Yablo's paradox rides again: a reply to Ketland

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Yablo's paradox is generated by the following (infinite) list of sentences (called *the Yablo list*):

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\begin{array}{l}
(s_1)\ \text{For all } k > 1,\ s_k \text{ is not true.}\\
(s_2)\ \text{For all } k > 2,\ s_k \text{ is not true.}\\
(s_3)\ \text{For all } k > 3,\ s_k \text{ is not true.}\\
\quad\vdots \qquad\quad \vdots \qquad \vdots\\
(s_n)\ \text{For all } k > n,\ s_k \text{ is not true.}\\
\quad\vdots \qquad\quad \vdots \qquad \vdots
\end{array}
$$

A little reflection reveals that this list is paradoxical. The source and nature of the paradox has been the focus of a fascinating debate. The crucial issue, of course, is whether Yablo's paradox involves circularity. Stephen Yablo (1993), Roy Sorensen (1998), and Bueno and Colyvan (2003b) have argued that the Yablo list generates a liar-like paradox without circularity. In the other camp are Graham Priest (1997) and JC Beall (2001), who argue that the paradox involves a fixed-point construction and therefore is circular. In Bueno and Colyvan (2003a), we respond by showing that there is a way of deriving a contradiction from the Yablo list without invoking any fixed-point construction and so, it would seem, the paradox does not essentially involve circularity.

In a recent paper, Jeffrey Ketland (2004) argues that our response is incorrect, and claims that the derivation presented in our paper is invalid. Ketland's paper raises some interesting issues; however, there also seems to be some confusion over what was going on in our paper. We take the opportunity here to clear up some misunderstandings and hopefully to clear the way for a better appreciation of the issues at hand.

Our aim in Bueno and Colyvan (2003a) was to show that, *contra* Priest (1997), a paradox can be generated from the Yablo list without applying the T-schema to open formulae and without invoking the satisfaction predicate. In response, Ketland attributes the following three claims to us and argues that we are mistaken on all three (2004: 167–8):

1. The introduction, by Priest, of the notion of satisfaction is unnecessary for the derivation of the paradox.
2. Priest's use of a fixed-point construction in the construction of the Yablo list is unnecessary.
3. The derivation of the paradox involves only what Ketland calls 'the local disquotation scheme': $T(\ulcorner Y_n \urcorner) \leftrightarrow Y_n$, with $n \in \omega$. (This principle is basically just the T-schema; it

asserts the equivalence of any specific (Yablo) sentence and the sentence asserting its truth.)

Let us consider each of these points of disagreement in turn.

In relation to (1), Ketland quite rightly points out that truth and satisfaction are interdefinable, but we never claimed that they weren't. The point of our discussion was to take up Priest's challenge to derive a paradox from the Yablo list, without illegitimate use of the T-schema on open formulae. We did not need to use the satisfaction predicate and what Priest (1997: 237) calls 'the generalisation of the T-schema to formulae containing free variables' (and Ketland calls 'the uniform disquotation T-schema'); we can do it all with a truth predicate and the garden-variety T-schema. The fact that satisfaction and truth are interdefinable is irrelevant. At the end of the day, it's the generalised T-schema that we wanted to avoid.

But perhaps Ketland's concern here is that since we used a truth predicate, and given the interdefinability of truth and satisfaction, we have not made any real progress; we've only dispensed with the generalised T-schema and the notion of satisfaction in a trivial sense. The argument to paradox can still be constructed, Ketland might complain, with the generalised T-schema and satisfaction. But we do not deny that paradox can be generated using satisfaction in the way that Priest and Ketland do. Our claim is simply that there is another way of deriving a paradox—a way that does not involve any fixed-point construction. The real issue is whether our argument to paradox succeeds using only the resources we allow ourselves. We return to this matter in our response to Ketland's third point of disagreement.

In relation to (2), Ketland runs together two quite separate matters, at least in his initial presentation of the issue, namely the construction of the Yablo list and the argument to paradox. It is the latter, we claim, that involves the fixed-point construction. Indeed, the passage from our paper Ketland quotes in relation to this point makes it clear that it's the *derivation* of the paradox that's at issue, *not* the construction of the list itself (Bueno and Colyvan 2003a: 153; Ketland 2004: 167). The matter of the construction of the list does not bear on this issue. In fact, we say that we take Priest to be presupposing the existence of the list and that the fixed-point construction involves only the argument to paradox (Bueno and Colyvan 2003a: 156, note 5).

Ketland complains. On his view, 'it is a *theorem of mathematical logic that the Yablo list exists*' (Ketland 2004: 169; italics in the original). But what is Ketland taking mathematical logic to be? Given the context, and his argument that the existence of the Yablo list follows from Peano arithmetic supplemented with a truth predicate, presumably he considers that combination to amount to mathematical logic. But, clearly, this combination is quite a bit more than mathematical logic unless one assumes, without argument, the truth of logicism! So it takes quite a bit more than mathematical logic to prove the existence of the Yablo list. Still, if the list is a theorem of Peano arithmetic supplemented with a truth predicate, isn't that enough?

Not really. We take the point that the existence of the list can be guaranteed in the way Ketland (2004: 169) rather nicely demonstrates.[1] But there is still the issue of whether such a construction is the *only* way to prove the existence of the list. Can't we construct the first few

[1] We also take it that this construction is what Priest (1997; and especially his comment in Bueno and Colyvan 2003: 156, footnote 5) had in mind, though never laid out so explicitly.

Yablo sentences by hand and then invoke mathematical induction to produce the rest? After all, each Yablo sentence s_n is exactly the same as the previous one with 'n-1' replaced by 'n'. So the construction of the list involves nothing more than knowledge of the natural numbers. Indeed, it would seem that under this construction, the Yablo list is no more circular than the natural numbers.[2]

The third complaint is the most substantial one. As Ketland notes, his main point (i.e., his argument against our claim (3) above) can be seen as follows:

> Each finite subset of Yablo biconditionals is satisfiable. By the compactness theorem, the whole set is satisfiable. (Ketland 2004: 165, note 1)[3]

Hence, Yablo's list is consistent after all! There is a very interesting mistake in this argument, though. While it is true that each finite subset of Yablo sentences is not *paradoxical*, it is not true that each subset is *satisfiable*. Consider, for example, the subset consisting of just the first two sentences:

(s_1) For all $k > 1$, s_k is not true.
(s_2) For all $k > 2$, s_k is not true.

Since there is no s_k for $k > 2$ in this set, s_2 is vacuously true. But this means that s_1 is straightforwardly false. That is, $\{s_1, s_2\}$ is not satisfiable. Note that $\{s_1, s_2\}$ is not paradoxical though: there is a consistent valuation function that assigns the value F to s_1 and T to s_2. Ketland seems to confuse unsatisfiability with paradoxicality. The latter implies the former but not vice versa. So, in the end, the compactness theorem is irrelevant here because not all finite subsets of the Yablo sentences are satisfiable. Hence, in particular, it does *not* follow by a compactness argument that the Yablo list is satisfiable and can only be ω-inconsistent. For all we know, the list might be simply inconsistent. Indeed, this point is crucial in identifying the mistake in Ketland's own more involved argument, to which we now turn.

According to Ketland:

> The list of 'Yablo biconditionals' (all instances of '$Y_n \leftrightarrow \forall m > n$, Y_m is not true') is *not* inconsistent with the relevant *local* disquotation principle (all instances of 'Y_n is true $\leftrightarrow Y_n$'). (Ketland 2004: 165)

But this is not correct. Note that each *instance* of a Yablo biconditional is a sentence of the form

(a) $Y_i \leftrightarrow \forall m > i$, Y_m is not true,

where 'i' is a natural number. Similarly, each instance of the local disquotation principle is a sentence of the form

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[2] We discuss the closely related issue of how to refer to the Yablo list in Bueno and Colyvan (2003b).
[3] Ketland attributes the point to a private communication with Stephen Yablo.

(b) Y_i is true $\leftrightarrow$ Y_i,

where ‘i’ is also a natural number. So, from (a) and (b), we get

(c) Y_i is true $\leftrightarrow$ $\forall m > i$, Y_m is not true.

Since each instance was arbitrary, we obtain from (c)

(d) For all n, Y_n is true $\leftrightarrow$ $\forall m > n$, Y_m is not true.

This is what Ketland calls ‘the Uniform Homogeneous Yablo Principle’ (Ketland 2004: 165), which, as he correctly points out, *is* inconsistent (2004: 167). But given that, as we have just seen, the Yablo biconditionals and the local disquotation principle *entail* the Uniform Homogeneous Yablo Principle, it follows that, as opposed to Ketland’s claim, the conjunction of the former two statements is also inconsistent.

But let’s grant, for the sake of argument, that Ketland’s reconstruction of Yablo’s paradox is consistent. It still *doesn’t* follow that the derivation of the paradox presented in Bueno and Colyvan (2003a) is invalid. In our derivation, we show that the truth of the *first* sentence in the Yablo list, namely

(s_1) For all $k > 1$, s_k is not true

entails a contradiction (Bueno and Colyvan 2003a: 154–155). We thus conclude that (s_1) is not true. This, in turn, we claim means that there is at least one true sentence in the Yablo list. Ketland disagrees. On his view, ‘one cannot *deductively* infer that “there is at least one true sentence in the Yablo list”’ (Ketland 2004: 171). Though Ketland fails to mention from what the existence of a true sentence in the Yablo list is supposed to follow, it is clear in our paper that the claim follows from the falsity of (s_1) (Colyvan and Bueno 2003a: 155). Indeed, (s_1) states that for all $k > 1$, s_k is not true; hence, its negation states that for some $k > 1$, s_k is true; that is, there is at least one true sentence in the Yablo list. (We call the first such sentence ‘s_i’.)

But Ketland seems to be concerned about whether the index k in s_k—where ‘s_k’ is a true sentence whose existence is guaranteed by the above argument—is a numeral standing for a standard or a non-standard natural number (Ketland 2004: 170). We had assumed, with everyone else in the debate so far, that it was a standard number. But if Ketland is right, and k might stand for a non-standard natural number, so what? The non-standard models of the natural numbers are also well ordered, and that’s all that matters here. There will still be a first true sentence in the list independent of whether the index ranges over standard or non-standard natural numbers. And our argument goes through either way.

Perhaps Ketland’s concern here runs a little deeper. Perhaps he is worried that only standard natural numbers correspond to Yablo sentences, that is, the non-standard natural numbers have no corresponding Yablo sentences. If this is his concern, he has given us no reason to doubt that non-standard numbers correspond to Yablo sentences, or, at least, that any substantive difficulty hangs on this. Why couldn't the Yablo list consist of sentences corresponding to standard *as well as* non-standard natural numbers? After all, as noted above, all that is required to set up the list is the ordering of natural numbers, whether standard or non-standard.

Ketland also takes issue with our interpretation of the subscript '*i*' in the first true sentence in the Yablo list, s_i.[4] We *stipulate* that *i* is *not* a variable and is an unknown particular natural number. Indeed, 's_i' is the name of a *particular* sentence, so '*i*' *cannot* be a variable. But according to Ketland:

> The '*i*' here is *either a free variable or a new constant*, and *not* (a numeral for) an 'unknown particular natural number'. (Ketland 2004: 171)

Given the options in the disjunction, of course we go for the new constant. In fact, that's what we took an 'unknown particular natural number' to be!

Ketland insists that '*i*' is a variable because, following Priest (1997), he wants to provide the most general argument to paradox from the Yablo list. And this does indeed involve a universal generalization. As Ketland puts it, 'we need to use [the index '*i*'] as a variable in quantificational reasoning with [the Yablo] sentences' (2004: 171, note 7). But, as noted above, in our derivation, '*i*' *is not a variable*, and deliberately so, given that the whole point of our paper was to bypass the need for the universal generalization invoked in Priest's original argument.

In conclusion, Ketland's papers (2004; forthcoming) provide an interesting reconstruction of Yablo's paradox. In particular, Ketland extends the framework developed by Priest (1997) and provides a neat analysis of the paradox. But none of what Ketland has to say undermines the main point of our paper, namely, that there is another, more modest, way to derive a paradox from the Yablo list. The way we suggest is modest in that it delivers paradoxical conclusions one sentence at a time. But we do this without invoking what Ketland calls 'the uniform disquotational T-schema'. Ketland, of course, is free to use whatever resources he wishes in deriving a paradox, and there's no denying that there is considerable merit in his way of looking at things. But, likewise, we are free to derive a paradox by more modest means, if we wish, and if we can do so legitimately. We stand by our claim that we achieved this in our original paper. We do not claim that ours is the only way of deriving a paradox from the Yablo list. We were, and are, content to show that Priest's way is not the only way to obtain a paradox from the list. In the end, paradox emerges from Yablo's list in multiple ways, and in at least one of these ways, no circularity is involved.[5]

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[4] Ketland complains about our notation as well (Ketland 2004: 171, note 7). But our notation is quite deliberately the same as Priest's (1997). We used Priest's notation so as to highlight the similarities and differences between his and our derivations of the paradox.

[5] Thanks to Laurence Goldstein for helpful discussion.

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