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YABLO'S PARADOX AND REFERRING TO INFINITE OBJECTS[1]

Otávio Bueno and Mark Colyvan

The blame for the semantic and set-theoretic paradoxes is often placed on self-reference and circularity. Some years ago, Yablo [1985; 1993] challenged this diagnosis, by producing a paradox that's liar-like but does not seem to involve circularity. But is Yablo's paradox really non-circular? In a recent paper, Beall [2001] has suggested that there are no means available to refer to Yablo's paradox without invoking descriptions, and since Priest [1997] has shown that any such description is circular, Beall concludes that Yablo's paradox itself is circular. In this paper, we argue that Beall's conclusion is unwarranted, given that (i) descriptions are not the only way to refer to Yablo's paradox, and (ii) we have no reason to believe that because the description involves self-reference, the denotation of that description is also circular. As a result, for all that's been said so far, we have no reason to believe that Yablo's paradox is circular.

I. Introduction

When looking for somewhere to place the blame for the well-known semantic and set-theoretic paradoxes, self-reference and, more generally, circularity are the usual suspects. In light of a remarkable paradox due to Stephen Yablo [1985; 1993], however, it is no longer clear that either circularity or self-reference should take the fall. Yablo has produced a paradox that's liar-like but does not seem to involve circularity. There has been considerable debate over whether Yablo's paradox is in fact circular.[2] If it is, then Yablo's paradox seems to lose much of its interest; it basically becomes a recently discovered member of a well-known family of circular and inconsistent constructions. However, if Yablo's paradox turns out *not* to be circular, that's significant indeed. After all, this fact would then provide an excellent case for the claim that circularity and self-reference are *not* necessary conditions for the semantic paradoxes. What this means is that the analyses of the semantic paradoxes that identify self-reference and circularity as the underlying 'causes' of the paradoxes fail to provide the full picture. There's more to semantic paradoxes than meets the eye. But is Yablo's paradox indeed non-circular?

1 We thank JC Beall for extremely helpful discussions and detailed comments on earlier versions of this paper. We are also indebted to both Roy Sorensen and Joel Stafford for discussion and comments on an earlier draft. We'd also like to thank an anonymous referee for many very penetrating comments and criticisms that prompted substantial improvements in the paper. The authors have changed their minds on many of the issues discussed in this paper several times. Such is the slipperiness of Yablo's paradox. All the views in this paper have been believed by each author at some time, but it is not the case that there is a time such that all the views expressed in this paper are believed by both authors at that time.

2 The main papers in the subsequent debate are Priest [1997] who argues that, despite initial appearances, the paradox is circular, and Sorensen [1998] who sides with Yablo against Priest.

This debate has taken a fascinating new turn recently. JC Beall [2001] has suggested, in effect, that there are no means available to refer to Yablo's paradox without invoking descriptions, and since Graham Priest [1997] has shown that any such description is circular, Beall concludes that Yablo's paradox itself is circular. A number of very interesting questions are raised by Beall's argument.[3] Here we will focus on two of these: (i) Are descriptions really the only way to refer to Yablo's paradox? And (ii) What reason do we have for believing that because the description involves self-reference, the denotation of that description is also circular? We suggest that 'no' and 'none' are the correct answers to these questions. If these are, indeed, the correct answers, Beall's argument for the circularity of Yablo's paradox fails, for it is essential to his case that (i) we cannot refer to the paradox by any means other than descriptions,[4] and that (ii) the paradox is circular if the only available means of referring to it is circular.

It might be useful to summarize Beall's argument as follows:[5]

(P1) Descriptions are the only way to refer to the set of sentences known as Yablo's Paradox (note that this singular term must therefore, presumably, be an abbreviation for such a description).

(P2) All descriptions of Yablo's Paradox are circular.

(P3) Any entity that can only be referred to by a circular description must itself be circular.

Conclusion: Yablo's Paradox, contrary to its surface appearance, is circular.

As will become clear, we contest premises (P1) and (P3).[6]

Next, we rehearse the paradox. Yablo's paradox is the paradox generated by the following denumerably infinite sequence of sentences:

(S_1) For all $k > 1$, S_k is not true.
(S_2) For all $k > 2$, S_k is not true.
(S_3) For all $k > 3$, S_k is not true.
. . .
. . .
. . .
(S_n) For all $k > n$, S_k is not true.
. . .
. . .
. . .

It's not difficult to see why the above sequence is paradoxical. Consider '(S_1)' in the sequence. Suppose '(S_1)' is true. It then follows that, for all $k > 1$, '(S_k)' is not true—and

3 We attribute the argument to Beall even though Beall suggests that he is merely clarifying the argument of Priest [1997]. We think Beall's contribution is more substantial than that, but nothing hangs on this.

4 By (i) it is not meant that Beall is committed to the *impossibility* of referring to the paradox via means other than descriptions; rather, Beall is simply committed to the view that we *do not* so refer, nor do we know of any way of referring to it but via descriptions.

5 We thank an anonymous referee of this journal for this way of laying out Beall's argument.

6 We contest (P2) elsewhere [Bueno and Colyvan 2003].

so, in particular, '(S_2)' is not true. However, if for all $k > 1$, '(S_k)' is not true, it also follows that, for all $k > 2$, '(S_k)' is not true. But this means that '(S_2)' is true. Given the contradiction, we conclude that '(S_1)' is not true after all. So, there is at least one true sentence in the sequence. Let the first such sentence be '(S_i)'. (Note that 'i' is not a variable, but an unknown, particular natural number.) Given that '(S_i)' is true, it follows that, for all $k > i$, '(S_k)' is not true—and so, in particular, '(S_{i+1})' is not true. However, if for all $k > i$, '(S_k)' is not true, it also follows that, for all $k > i+1$, '(S_k)' is not true. But this means that '(S_{i+1})' is true. Contradiction (since we have already established that '(S_{i+1})' is not true). Thus, the Yablo sequence is paradoxical and, on the face of it at least, seems not to be circular (no sentence refers either to itself or to sentences above it in the list).

II. The Case Against Demonstrations

A. The First Move: The Infinity of Yablo's Sequence

Crucial to Beall's argument for the circularity of Yablo's paradox is the claim that we have no means available to refer to Yablo's paradox without a (self-referential) description.

> Everyone, I think, will agree: we have not fixed the reference of 'Yablo's paradox' via demonstration. Nobody, I should think, has seen a denumerable paradoxical sequence of sentences, at least in the sense of 'see' involved in uncontroversial cases of demonstration.
>
> [Beall 2001: 179]

The point Beall is making here is not one about the abstract nature of the paradox (or of the sentence types that constitute it). Rather, Beall's concern is that we cannot fix the reference via demonstration *because the paradox is infinite*. The fact that the paradox consists of an infinite list means, according to Beall, we cannot effect the baptism required for a demonstrative reference fixing.

We disagree. In this section, we will show how demonstration can be used to fix the reference of 'Yablo's paradox'.[7] Beall, himself, considers one such approach to demonstrative reference fixing:

> Perhaps one might protest that we have seen Yablo's paradox; we have seen tokens of some of its constituent parts . . .; and seeing this much, the suggestion goes, is sufficient for the required 'baptism', which thereby affords fixing the reference of 'Yablo's paradox'. This suggestion, while interesting, cannot be maintained easily, at least not without much more explanation. To begin, there are infinitely many distinct sequences whose first few tokens appear exactly as [in the usual presentation of Yablo's paradox, as above]. Which one is being baptised?
>
> [Beall 2001: 179–80]

[7] Indeed, Beall concedes that if we can refer to the paradox via some other means (other than descriptions), his argument does not go through: 'I grant that if the reference of "Yablo's paradox" can be fixed (uniquely) without recourse to description, then the case for circularity crumbles quickly' [Beall 2001: 185–6].

Here Beall argues that we cannot refer to Yablo's paradox via demonstrations because the initial finite segment that we have access to underdetermines the complete infinite list. But if this were correct, we would not be able to refer, via demonstrations, to *any* infinite sequences. In particular, we would not be able to refer, via demonstrations, to the natural numbers. Now, clearly we do successfully refer to the natural numbers and equally clearly we successfully refer to them via descriptions. The question then is whether we can refer to them via demonstrations as well. For if we do refer to the natural numbers via demonstrations, this would provide a counterexample to Beall's claim.

But we do both learn about the natural numbers and refer to them by demonstrations. There simply is no problem knowing what the next natural number is, given any initial sequence.[8] The sequence of natural numbers is far simpler than other sequences where underdetermination worries clearly arise. Consider, for example, the sequence: '3, 5, 7, . . .'. The next member of this sequence is not determined by the first three members. After all, is this the sequence of odd primes or the sequence of odd numbers greater than one? If the former, the next member is 11; if the latter, it's 9. But no such subtleties are present in determining the next member of the sequence: '1, 2, 3, 4, 5, 6, 7, . . .'.

Well, perhaps we're a bit quick here. Hasn't Kripke [1982] taught us to be suspicious of even simple arithmetic operations like addition and the successor function? But such suspicions are not what Beall has in mind; Beall is not mounting any general sceptical challenge of this sort. His concern is that there is a problem here in the case of the Yablo sequence (and perhaps some other sequences). But what is the problem? An examination of the initial segment of the Yablo sequence reveals that working out what the next sentence is, is simply a matter of working out what the next natural number is. The Yablo sequence is very much *unlike* the clearly underdetermined sequence: '3, 5, 7, . . .'. So, short of general sceptical worries, there seems no reason to think that Yablo's sequence is underdetermined.[9]

It might be argued, however, that there's an important disanalogy between the generation of the natural number sequence and the sentences in the Yablo list. The rule for generating the next natural number depends only on numbers already generated. Whereas, in the Yablo list, each sentence refers to later sentences—sentences not yet generated—and so each sentence in the list depends on sentences not yet generated.

In response to this objection, we first note that the generation of each sentence *token* certainly does not require the generation of later sentences. The generation of the tokens requires nothing more than knowledge of the natural numbers. After all, the only difference between sentence i in the list and sentence i+1 is the substitution of 'i+1' for 'i' in the sentence in question. Nothing other than the knowledge that i+1 is the successor of i is required for this. So surely this is not the worry expressed in the above objection.

[8] The way the sequence of natural numbers is normally introduced to those who do not know it crucially depends on demonstrations. We are first acquainted, via demonstration, with the basic sequence: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Using this sequence, we then learn a procedure for generating subsequent natural numbers. We learn to go back to 0, add a 1 to the left, and go through all the sequence on the right digit. We then replace the 1 by its successor 2 and go again, and so on. Each stage in this process depends on the ability to identify the terms in the basic sequence, and so demonstration is needed throughout the process.

[9] We should also point out that general sceptical worries arise not only in relation to infinite sequences; reference to finite lists suffers the same problem. Consider the underdetermination present in the following list of five natural numbers: '3, 5, 7, . . .'.

Perhaps, instead, the worry is that knowing the meaning of each sentence in the list requires the generation of the whole list. But this is not right either. We clearly understand the meaning of each sentence in the list. To see this simply note that in order to determine the truth or falsity of some sentence in the list, we recognize that we need to look at others below it. How would we know this if we didn't already understand the meaning of the sentence in question?

So there's no problem generating the list of sentence tokens, nor is there any problem with grasping the corresponding meanings. The only thing left that might be problematic is the fact that each sentence in the Yablo list refers to other sentences not yet generated. How, then, can we ever get started, you might ask. Put thus, the worry is that we cannot generate a sentence containing a term whose referent is not yet generated. But consider statements about the future. At least on some views about time, the future does not exist. This, however, does not mean that one cannot generate statements about the future. As for the meaningfulness of such statements, this will depend on your theory of meaning. Typically, however, such statements *will* be taken to be meaningful. Some take all such statements to be false or without truth value, but they are at least meaningful. So too, we say, for the Yablo list: each sentence can be generated, each sentence is meaningful, and that's all we need for the moment.[10]

B. The Second Move: Demonstration Or Description?

There is another, subtler, reason to be unhappy with Beall's argument. Beall seems to assume that there is a sharp distinction between fixing the reference of 'Yablo's paradox' via description and via demonstration. As Beall points out:

> We basically have two ways of fixing the reference of [a term] *t*: Demonstration or (attributive) Description. . . . What is important for present purposes is the question: Have we fixed the reference of 'Yablo's paradox' by the first method (demonstration) or the second (description, attributive)?
>
> [Beall 2001: 179]

In other words, Beall seems to assume that the description method involves *only* descriptions and the demonstration method involves *only* demonstrations. But this does not seem right.

Consider, first, how we fix the reference of 'natural number' by the description method. The *standard description* (or *standard characterization*) of the natural numbers is the following:

(1) 0 is a natural number.

(2) For every x, if x is a natural number, then so is $x + 1$.

[10] We do, however, acknowledge that there is a difference between the natural numbers and the Yablo list. The natural numbers are predicative, whereas the Yablo list, one might argue, is impredicative. This raises serious issues about the relevant sense of circularity here: is it or is it not impredicativity? We take up this matter in Section III, where we argue that if one were to accept impredicativity as the relevant sense of circularity, one would be lead to accepting unintuitive results elsewhere. For example, it would follow that real numbers would be circular.

But note that although (2) is clearly a descriptive clause, (1) is a demonstrative clause. It picks out the object zero and tells us that *this* is a natural number. What we mean by 'picking out the object zero' is this: in an inductive definition of a class N (such as the class of natural numbers), the base clause needs to be fulfilled by an object that is assigned as an element of N. The *nature* of the object that plays this role is not crucial, but only that *there is* such an object. We pick out *zero* as *this* object—and in this sense, (1) is a demonstrative clause.[11] So, we see that the standard 'description' of the natural numbers involves both descriptions and demonstrations. Moreover, without the first, demonstrative clause the second can be vacuously true, in which case the 'description' is empty. So, if this description successfully refers to the natural numbers, it does so because of the demonstrative clause (1).[12]

But now the question arises: if demonstrations play a role in the characterization of natural numbers, what is the *demonstratum*? Is it a Fregean object, a Zermelo set, a von Neumann ordinal, an ink mark—or something else altogether? Of course, the answer 'It's a natural number!', although not incorrect, is not at all informative, given that 'natural number' is exactly what we are trying to characterize. But this latter answer *doesn't* entail that the natural numbers are circular. All that is required for the inductive definition to work is that we have infinitely many objects with a successor function defined on them. We pick out one of the objects, call it '0', and proceed as above.

Exactly the same point applies to Yablo's paradox. The analogue of (2) above in describing the Yablo sequence of sentences is (S_n). This is the descriptive clause, while (S_1)–(S_3) are demonstrative clauses. (S_1)–(S_3) are demonstrative clauses in the sense that sentence number i picks out the number i and says that all sentences whose number is greater than *this* number (that is, greater than i) are not true. (S_n), on the other hand, is not, strictly speaking, a member of the list (because n is not a natural number, it's a variable ranging over natural numbers). (S_n) is a *description* of the form of all the sentences on the list. So, in effect, we start out by demonstrating members of the sequence but since we can't list them all, we need a description to ensure that we pick out the correct continuation. Thus (S_n) plays the same role here as (2) does in picking out the natural numbers. And just as (2) won't deliver the natural numbers on its own, so too (S_n) does not result in paradox on its own. After all, (S_n), *on its own*, is vacuously true. There's nothing paradoxical about that. We need at least (S_1) to ensure that (S_n) is not vacuous, just as we need (1) to ensure that (2) is not vacuous. In short, at least one demonstrative clause, such as (S_1), is essential for the generation of the paradox.

It might be complained that there is an important difference between the natural numbers and the Yablo list and that this difference undermines the above analogy. The difference is that the generation of the next natural number depends only on the natural numbers already generated, whereas each sentence in the Yablo list depends on later members of the list. But this, as we argued in the previous section, is not the case. We do

[11] Note that (1) cannot be a descriptive clause—otherwise the standard characterization of the natural numbers would be circular. For we need clause (1) to be *true*—true of something—for the inductive characterization of N not to be vacuous.

[12] In any case, even if you're not convinced that (1) picks out zero by a demonstration, it is clear that (1) does not pick out zero by description. That's all we really require here: reference to the natural numbers *via the standard characterization* will not succeed without a non-descriptive element. We take that non-descriptive element to be a demonstration, of sorts, but nothing hangs on this.

not need the later sentences to generate a particular sentence token, nor do we need the later sentences to understand the meaning of any previous sentences. At any rate, the difference between the natural numbers and the Yablo list mentioned above is not relevant here. The point we are making is that in both cases, descriptive and demonstrative clauses are required to fix the reference. In that sense the natural numbers and the sentences in the Yablo list are analogous.

Now, Beall's argument requires that the reference of 'Yablo's paradox' be fixed by descriptions and descriptions alone. But if what we've suggested above is correct, the reference is fixed via *both* descriptions and demonstrations. And, most importantly, it seems that *both are required for the reference fixing.* Thus, Beall's argument simply does not apply to Yablo's paradox.

Now, it might be objected that the important issue wasn't whether we *could* know what the Yablo sequence is; rather the issue is *how* we know it. And, the objection continues, we know what the next Yablo sentence is because we know the description of Yablo's sequence; but its description, as Beall argues, is circular (as Priest's [1997] fixed-point theorem attests). So Beall could agree that we know what the next Yablo sentence is; we know it via the general circular specification of Yablo's sequence.

We grant that the issue is *how* we know the Yablo sequence. And our response is: we know the sequence exactly in the same way as we know the natural numbers—i.e., via the standard characterization, which is constituted by a base clause and an inductive clause. The latter clause is clearly a description, and so the former *cannot* be. For if the base clause were a description, the characterization of the natural numbers would be circular. We would need to presuppose that *we already know what 'natural number' is* to be able to say that (1) 0 is one of them, and (2) if x is one of them, so is $x + 1$. Our point is: for this characterization to get off the ground (1) needs to *pick out an object*. *Which* object? *That* object, we say—the first in the sequence to be characterized. In *expressing* the point we need to invoke something akin to a description; but ultimately the base clause is *not* a description.

It might be argued that even if we pick out Yablo's sequence by demonstration *and* description, this fails to block Beall's argument. As long as a circular description is necessary (even if it is not sufficient), Beall's argument still gets under way. But this is not the case. First, as the analogy with the natural numbers indicates, the *demonstrations* required to refer to Yablo's paradox don't establish that the latter is circular, any more than the demonstrations required to refer to the natural numbers establish that such numbers are circular. Second, as we will see in the next section, for Beall to conclude that Yablo's paradox is circular from the circularity of the *description* of the paradox, he needs to assume a principle, which we call the *general claim*, that turns out to be false.

III. The Case for Circularity

Let us grant, for the sake of argument, that we refer to Yablo's paradox by descriptions and by descriptions only. Beall claims that if the description is circular and there is no alternative method of referring, then the object referred to is also circular. We need to be careful about what Beall's point here is. It is not that the properties of the description are 'transferred' to the referent.

> The point, rather, is that any description, *D*, used to fix the reference of 'Yablo's paradox' is such that *D's* satisfaction conditions require that the (unique) satisfier is circular (contains self reference, a fixed point, etc.).
>
> [Beall 2001: 184]

First, note that Priest's [1997] fixed-point theorem is, ultimately, about the (circularity of the) *descriptions* of the Yablo sequence. It establishes that any such description has a self-referential fixed point. On the face of it at least, it doesn't establish the circularity of the *satisfiers* of those descriptions. Sorensen [1998] recognized this gap, pointing out that even though our description of Yablo's paradox may indeed be circular, this doesn't provide any reason to claim that the actual paradox (the sequence so described) is circular. In response, Beall provides an argument to fill this gap. In short, Beall's argument is supposed to take us from Priest's theorem and the talk of circular descriptions to the circularity of the satisfiers of those descriptions. The only condition Beall requires for this move is that there be no other method of fixing the reference. That is, he is committed to the following *general claim*: there are no non-circular objects that we can denote only via circular descriptions.[13]

But it turns out that *there are* such non-circular objects. Consider the supremum (or least upper bound) of subsets of real numbers:

> DEFINITION (Least Upper Bound): The least upper bound s of a subset σ of real numbers is the smallest number that is larger than every member of σ.[14]

Note that this definition is circular in that we only successfully refer to s by referring to each member of σ. Moreover, this is the *only* way to refer to some (non-algebraic) real numbers in the context of classical analysis.[15] This is because the real numbers are the smallest set that contains all the accumulation points of series of rationals. And since almost all these real numbers are non-algebraic and few have convenient names (such as e and π), the only way to refer to the rest is via some Cauchy sequence for which the number in question is a supremum (or infimum) [Simmons 1963: 70–5]. (Alternatively, we can refer to these numbers as Dedekind cuts of rationals, but the idea is much the same [Rudin 1976: 17–21].) So, there are some real numbers whose only description is circular. But the supremum is a real number and, we take it, clearly *not* a circular object. Thus the general claim above is false and Beall's argument for the circularity of satisfiers does not go through.[16]

[13] Talk of circular *objects* may sound confused, or even a kind of category mistake (unless, for example, we're talking about geometric properties, which is not the case here). We return to this issue later in this section, but for now we're content to follow the usage of others in the debate, though we admit some uneasiness about such talk.

[14] More formally, we say that a real number s is the supremum of a subset σ of real numbers if it satisfies two conditions: (i) $u \leq s$ for all $u \in \sigma$, and (ii) if v is any number such that $u \leq v$ for all $u \in \sigma$, then $s \leq v$.

[15] There have been various attempts (most notably by Weyl [1918]) to provide predicative definitions (that is, non-circular definitions) of concepts such as 'supremum'. These attempts fail to deliver the expressive power of the standard definitions of the concepts in question.

[16] You might think that the above general claim is too strong, and that all Beall requires is that there be a descriptive element in the fixing of the reference of Yablo's paradox. If this descriptive element is circular, then Beall's argument still goes through. But this won't help. After all, the counterexample considered above also serves as a counterexample to this weaker claim: the supremum has a descriptive element in its definition and yet is not a circular object.

But wait a minute. What is the sense of circularity at issue here? Isn't it just impredicativity in the previous example, rather than 'genuine' circularity? There are, of course, several senses of circularity in the literature. But for our purposes here, there is no need for reviewing them all.[17] It's enough to note that the sense in which the characterization of the Yablo sequence is claimed to be circular is ultimately the sense in which one could claim that the notion of supremum is circular. It might turn out that, like the real numbers, the Yablo sequence is impredicative. What this means is that the characterization of a member of Γ—whether this member is the supremum of a subset of real numbers, or a term in the Yablo sequence—can only be made in terms of Γ itself. In the case of the supremum s of σ (where, as above, σ is a subset of real numbers), s is characterized (in part) as being larger than *every member* of σ. And so, the whole set σ is presupposed to characterize s. Similarly, the characterization of the generic term in the Yablo sequence, namely

(S_n) For all $k > n$, S_k is not true,

also involves reference to *all subsequent terms in the sequence*, that is, to *all* (S_k) such that $k > n$.[18] Our point is simply that given that we *don't* think that real numbers are circular, we have no reason to think that the terms in the Yablo sequence are circular either.

Now the question arises: even if each term in the Yablo sequence is non-circular, does it follow that the *whole sequence* is also non-circular? Of course, there are well-known cases of sequence of sentences in which although the terms in the sequence aren't circular, the whole sequence clearly *is*. Liar cycles provide an obvious example:

(a) Sentence (b) is false.

(b) Sentence (a) is true.

But the structure of the Yablo sequence is *very different* from that of the above cycle. After all, *no* sentence in the Yablo sequence ever refers to any sentence above it in the sequence, and no sentence in the sequence ever refers to itself. Thus, nothing in the description of the whole Yablo sequence entails its circularity.

But there is no need for us to push the analogy any further here. Our point with the supremum example is simply to establish the falsity of Beall's *general claim*. There *are* non-circular objects (namely, the supremum of a subset of real numbers) that we can denote only via circular descriptions. And without the general claim, Beall's argument is blocked, given that there is no warrant to move from the circularity of the description of Yablo's paradox to the circularity of the satisfiers of this description.

Finally, let's return to the issue of the relevant sense of circularity. There has been a certain amount of unclarity about this issue in the debate over Yablo's paradox. There are at least two that might be relevant here:[19]

(C1) The descriptions of the set of sentences is circular (e.g., impredicativity, the existence of fixed points, etc.).

[17] We'll return to this issue at the end of this section.

[18] Given that the Yablo sequence is infinite, it's a moot point to say that we are not referring to the whole sequence given that an initial segment is being disregarded.

[19] We thank a referee of this journal for the following distinction.

(C2) The pattern of reference displayed by the sentences is circular.

Now clearly the Yablo sequence is not circular in the sense of (C2), but Priest [1997] argued that it is circular in the sense of (C1). The question then raised, by Beall [2001] and Sorensen [1998], is whether *the sequence itself is circular*. While in this paper we have argued that it is not, we admit some puzzlement over what is at issue here when we, and others, talk of the circularity of *the sequence itself*. Indeed, does it even make sense to talk of a 'circular object' (except in the obvious geometric sense)?

Perhaps the only sense we can make of circularity is in terms of (C1) or (C2). But even here there has been some slippage. Previously when people talked of the liar or liar cycles being circular, they meant that there were *referentially circular*. That is, they meant that they were circular in the sense of (C2). The recent debate about Yablo's paradox has focussed on circularity in the sense of (C1). But already this is to admit a big difference between this paradox and the other liar-like paradoxes: the latter are referentially circular, while Yablo's paradox is not. If Yablo's paradox turns out to be circular in some other sense (such as in the sense of (C1)), that may be interesting but it's somewhat beside the point. The fact remains that Yablo's paradox is not referentially circular, and that, after all, was all Yablo originally claimed. Moreover, it was this claim of Yablo's that Beall and Priest are supposed to be contesting. In this paper we have taken issue with Beall's particular path in this debate.

IV. Conclusion

Where does this leave the debate? Yablo has presented a paradox that appears to be liar like but without circularity. Priest has argued that, despite appearances, the paradox *is* circular. Sorensen resisted this claim by pointing out that there is a lacuna in Priest's argument. Beall's contribution to the debate was to defend Priest's argument in the face of Sorensen's criticism. In particular, Beall's paper was supposed to show how we get from the circularity of the description to the circularity of the satisfier.

What we have argued in this paper is that Beall's argument fails on two fronts. First, there are other means (namely demonstrations) available for fixing the reference of 'Yablo's paradox'. And, in any case, even the standard description of the Yablo sequence is not *purely* a description—it employs demonstrations as well. Second, even if pure descriptions were the only way to refer to Yablo's paradox, there are counterexamples to the move from the circularity of the description to the circularity of the satisfier. So, for all that's been said so far, we have no reason to believe that Yablo's paradox is circular. Hence, in the diagnosis of the semantic paradoxes, far more work needs to be done before we can blame circularity and self-reference for the mess.

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