The Limits of Subtraction

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1. Introduction

Very often in philosophy and in every-day life we say things that we do not hold to be literally true. I have in mind here statements such as (i) “even God couldn’t know the extension of a vague predicate” and (ii) “she’s struggling with her own personal demons ”. Statements such as (i) and (ii) do not commit the speaker to gods or demons. What is intended in each case is clear enough but were there any confusion, the speaker could make the non-commitment to gods and demons clearer by retracting any such apparent commitment.

There are a couple of ways of executing the retraction. One is to say directly what was intended in the first place by paraphrasing the sentence in question to deliver the intended content and nothing more than the intended content. In place of (i) we would have something like (i*) “The extension of a vague predicate is in principle unknowable” and in place of (ii) we would have something like (ii*) “She is currently experiencing some serious personal problems”.

Actually, the translation of (ii) is not so straightforward. Depending on context, (ii) might be thought to imply that she has some mental health issues, she is suffering from drug addiction or she has serious trauma in her past. So despite the obvious virtues of attempting to say more directly what is meant via (i*) and (ii*) this is not always possible. It is very hard to preserve the ambiguity of (ii) with any translation such as (ii*). This brings us to the next strategy for retracting commitments to gods and demons: *subtraction*. This strategy is the focus of Stephen Yablo’s target paper (forthcoming) and will be the main focus of this paper.

The idea of subtraction is that one simply subtracts the unwanted ontological commitments from the original statement. So in place of (i) we have (i**) “even God couldn’t know the extension of a vague predicate (except for the stuff about god)” and in place of (ii) we have something like (ii**) “she’s struggling with her own personal demons (except for the stuff about demons)”. We thus attempt to express the godless and demonless content without paraphrasing.

As we have already seen, subtraction has a significant advantage over the paraphrase strategy in that sometimes there may not be a paraphrase ready to hand and yet we can still achieve our goal by subtraction. Perhaps the most significant case of interest here is that of applied mathematics. Here we have statements about the physical world expressed in mathematical form and explicitly quantifying over mathematical objects. Nominalists do not want to take the mathematical content of such statements seriously. At least they do not want to take the mathematics *ontologically* seriously; they take the mathematics seriously in so far as the mathematics helps convey how the physical world is—the nominalisitic content.

Many philosophers have advanced nominalistic proposals along these lines[1] but recently Yablo's has developed a very clear and compelling case for subtraction and spelled out a possible worlds account of the technical details.[2] Rather than focus on the details of Yablo's account—as interesting as those are—here I want to step back and look at the very idea of subtraction. My aim is to clarify the strategy and to see its limitations by appealing to an analogy with photographic subtraction. In general terms, though, Yablo is interested in φ except for ψ or, equivalently, the remainder when ψ is subtracted from φ. He uses the notation $\varphi \sim \psi$ for this remainder.

2. Photographic Subtraction

In photographic subtraction there are two images: the first let's call the *entangled image* φ and it has what we are interested in and more besides. We also have a *mask image* ψ, which is an image of that which we are not interested in—but in exactly the same position and orientation as the entangled image. If we make a negative of the mask and overlay it with the entangled image (both as transparencies), we end up with an image of just that which we are interested in φ minus ψ (or $\varphi \sim \psi$, in Yablo's notation). That is we have an image of φ except for ψ. Examples here include radiographic subtraction during cerebral angiography, where the mask is a plain skull x-ray and it is used to subtract the bony detail from the angiogram itself to produce a clear image of the blood vessels of the brain (highlighted with a contrast agent injected into the arterial system) free from overlying bony structures. Such subtraction is typically carried out digitally these days but the principle is exactly the same.

Photographic subtraction is a nice model of the kind of ontological/semantic subtraction Yablo is interested in. It clearly makes sense and it works. It is important to note, however, what is required to make it work. We need a mask in the photographic case (and an independent grip on ψ in Yablo's account). The mask must be distinct from the entangled image; the entangled image cannot be used as a mask otherwise there is nothing of interest left. Moreover, these enabling conditions are non-trivial and can easily fail to be met.

Finally, a point of disanalogy with the photographic case. In photographic subtraction, if everything goes according to plan, the existence and uniqueness of $\varphi \sim \psi$ is assured. It might be empty or otherwise trivial (e.g. uninteresting) but there will be a $\varphi \sim \psi$. In other cases under discussion, existence and uniqueness cannot be taken for granted. In particular, we need some reason to expect that there will be a unique (or at least not massively underdetermined) image as a result of the subtraction. As I've argued elsewhere, you cannot subtract entities that are too entangled. For example, you cannot expect to subtract hobbits from

[1] See, for example, Balaguer (1998) and Melia (2000), and Burgess and Rosen (1997) for a systematic survey of nominalist strategies for mathematics.

[2] The primary account here is found in Yablo (forthcoming) and (2014) but he has been pushing in this direction for some time with his work on metaphor (1998, 2005, 2012). See also Mary Leng's work along similar lines explicitly aimed at developing a fictional account of mathematics (2010, 2012, forthcoming).

The Lord of the Rings (Colyvan 2010). While there may well be something in the remainder (i.e. existence is satisfied), it is so massively underdescribed that it is of no interest (i.e. we do not have uniqueness). For example, we do not know who carried the ring and why they were not corrupted, we do not know whether Hobbiton is still in The Shire and so on. The point is that hobbits are simply too central to the story to be subtracted, even though we may have a clear sense of what our mask ψ is.

Another way of pressing this point is to note that hobbits play an explanatory role in the story. They are not simply part of the scenery. The latter, presumably, could easily be subtracted, but if we subtract hobbits, we lose crucial parts of the story itself and, in particular, we lose explanations for crucial events in the story. This point generalizes: if, as in science, it is explanations we seek, we need to be careful not to subtract away our explanatory resources.[3]

3. Explanatory Entanglement and Applied Mathematics

Now return to the case of applied mathematics. The important question is whether mathematics is subtractable in the way that God can be subtracted from (i) and demons can be subtracted from (ii), or whether we are dealing with something more like the hobbit case. Although not couched in these terms, this is precisely where the debate about the indispensability argument in the philosophy of mathematics currently stands. The central issue is whether mathematics plays explanatory roles in science. If it does, mathematical realism beckons; if not, it looks as though some kind of subtraction strategy[4] might work.

On one side of this debate are those who accept that mathematics plays an indispensable role in science but argue that this role is one of mere indexing (Melia 2000). According to this view, mathematics does nothing more than stand proxy for the physical stuff of the universe and it is the latter that does the explaining. Thus, "there are seven deadly sins" is about the sins, and the numbers 1 through 7 are just used to index the sins. On the other side of this debate are those such as Alan Baker (2005) and me (Colyvan 2001, 2010) who argue that in some cases mathematics does more than merely index the physical world; the mathematcs in such cases is indispensable to scientific explanations. To gain purchase on this debate, a simple example might help.

The Intermediate Value Theorem of elementary calculus can be used to explain many things.[5] For example, this theorem can be used to explain why you need to

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[3] In the cerebral angiogram this would be like noticing a displacement and compression of the blood vessels on one side of the brain (in fact due to a skull fracture) but not being able to explain this because the bony detail has been subtracted away. If it's skull fractures we're interested in, we need the bony detail.

[4] Or similar so-called easy road nominalist strategies (Colyvan 2006, 2010, 2012). These are strategies that do not require the difficult task of producing a mathematics-free science as Hartry Field attempted in *Science Without Numbers* (Field 1980). We should also question whether the mathematical content and the empirical content are neatly separable in the way that subtraction requires. This too is non-trivial to demonstrate.

[5] Let f be a real-valued function, continuous on a closed interval [a,b] and let c be any number between f(a) and f(b) inclusive, then there is an x in [a,b] such that $f(x)$=c.

cross the equator when travelling north from Sydney to Tokyo and why if the temperature is 10 degrees Celsius in the morning and 20 degrees Celsius early in the afternoon, there must be a time between when it is 15 degrees Celsius. The explanation in both cases is essentially the same and yet any mathematics-free account will fail to deliver this. After all, mathematics-free accounts will focus on geography and the weather respectively but the similarity between the cases is only seen from the abstract perspective offered by mathematics. Moreover, it is hard to see how mathematics-free explanations could recognise the modal force of the explanation. The mathematics-free explanation runs the risk of making it contingent that there is such an equator crossing or a time when the temperature is 15 degrees Celsius. But in both cases the phenomena in question are necessary. For these reasons, I'm inclined to say that mathematics is playing an explanatory role in such cases.[6] Nominalists, of the indexing variety, at least, must maintain that mathematics is simply playing an indexing role and the *real* explanatory work is being done by the geography and the weather.

4. Conclusion

I like the subtraction idea—it clearly has merit. But just as with photographic subtraction we need to exercise some care in its application. Indeed, this in itself is good reason for the process to be spelled out in detail as Yablo does so admirably (forthcoming). It is not enough to simply assume that we can subtract mathematics from science and that there will be a meaningful remainder: *the nominalistic content*. None of this is news to Yablo, who is well aware of the limitations of subtraction. Indeed, Yablo suggests that whether this strategy succeeds "will depend on, among other things, how much of interest is left when ψ, is subtracted from φ'' (Yablo, forthcoming). It is worth mentioning, though, because it is all too easy to skip over what is required for any such subtraction approach to succeed.

There seems to be a common thought pervading much of the debate over easy-road nominalist strategies for applied mathematics, namely, that with or without mathematics, the world would look the same. That is, subtract the mathematics and everything the nominalist cares about would be the same. As Mark Balaguer puts it ''the physical world holds up its end of the 'empirical-science bargain' " (Balaguer 1998, p. 134). But such a partition of the world into nominalstic content and mathematical content is problematic, and it is anything but clear that there is anything of interest in the nominalistic content. We might simply end up with something like the hobbitless *Lord of the Rings*.[7]

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[6] There is also the interesting issue of what kind of explanatory role mathematics plays. See Yablo (2012) and Colyvan (2012) for more in this.

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