Tolerating Gluts

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In an approach to vagueness using the paraconsistent logic LP, borderline cases of vague predicates are contradictory — logical *gluts*.[1] In 'Finding Tolerance without Gluts' (Beall 2014),[2] Jc Beall argues against such an account of vagueness. He constructs an alternative theory, and argues that '[t]he result enjoys all the virtues of the LP solution but without the gluts' (p. 794). He concludes that his alternative is therefore preferable to the LP solution.

In what follows, we will demonstrate that this is not the case: Beall's account does not do all the things that a paraconsistent solution can do. In fact, it is the other way around: the paraconsistent account can do everything that Beall's theory can do, and more. And some of the 'more' is *very* important. We will demonstrate this by discussing each of the three objections to his own project which Beall raises and rejects (Sects 4.1, 4.3, 4.5), arguing that his replies fail in each case.

This note is not solely a reply to Beall. Several quite new points emerge in the discussion, especially in section 2, clarifying the paraconsistent account.

[1] As far as we know an LP-solution to the sorites paradox was first advocated by Ripley (2005) following conversations with Beall and Colyvan in 2004, and by Hyde and Colyvan (2008). More recently we have Priest 2010, Weber 2010b, and Ripley 2012. Paraconsistent solutions to the sorites, of course, go back much further. Some of the earliest paraconsistent logics were seen as logics of vagueness (e.g. Jaśkowski 1948 and Halldén 1949). The revival of paraconsistent approaches to vagueness was sparked by Hyde (1997).

[2] Unless otherwise indicated, page and section references are to this.

Mind, Vol. 123 . 491 . July 2014
doi:10.1093/mind/fzu057 Advance Access publication 6 October 2014

1. Versions of tolerance

The approach to vagueness using the logic LP has at least this virtue: it respects the pairwise equivalence of consecutive claims in a sorites sequence, which is a highly intuitive feature of any such sequence. Beall's main idea is to transfer this feature without loss to an alternative framework. The difference, says Beall, is in the particular pairwise equivalence connection allowed by the two theories. An LP-based theory takes the relevant sort of pairwise equivalence to be that encoded by a material biconditional. Beall's unassertibility-based theory, on the other hand, takes it to be encoded by the biconditional-like connective built around an 'unassertibility operator', μ, and its associated conditional-like connective $\sqsupset$, defined so that $A \sqsupset B$ is $\mu(A) \vee B$. We refer the reader to his paper for full details of the approach.

Beall employs a nonstandard and possibly misleading notation for these two biconditionals. Beall writes $\equiv_{lp}$ for the material biconditional and $\equiv$ for his unassertibility-based connective. But $\equiv_{lp}$, as he defines it, is simply the familiar material biconditional: $A \equiv_{lp} B$ is defined as $(\neg A \vee B) \,\&\, (\neg B \vee A)$. Note well: despite the subscript, there is nothing distinctly LP-ish about this. LP-based theorists have a distinctive theory of *negation*, but the material conditional and biconditional are the familiar ones; the conditional is not some newfangled connective, or a connective distinctive to the logic LP. Accordingly, we write $\equiv$ for the material conditional, and we write $\equiv_{Beall}$ for Beall's unassertibility-based connective.

The connective $\equiv$, on an LP-based theory, and $\equiv_{Beall}$, on anyone's theory, share some features that make them appealing for the purpose of capturing the pairwise equivalences that occur in a sorites series, as Beall points out. Key amongst these are reflexivity, symmetry, and nontransitivity. Since these features are shared, the approaches do indeed share some common ground. But Beall's claims for the unassertibility-based approach — in particular, the claim that it enjoys all the virtues of the LP-based approach — cannot be sustained.

Let us start by considering a core objection: Beall's approach has nothing to say about versions of the sorites that *do* use the material conditional. (See Beall's Sects 4.5, 4.6.) In standard formulations of the sorites paradox, the major premisses are conditionals — or the corresponding biconditionals. *Every* biconditional connective will generate a version of the sorites argument. There is one, in particular, for the material biconditional, $\equiv$. A solution is required for this. Beall proposes his own conditional, $\equiv_{Beall}$. Even if we grant him everything he says about this conditional and the sorites formulated in terms of it (which we will not), this is *irrelevant*. We have proposed a solution to the version of the paradox with $\equiv$. He discusses a different version of the paradox. As such, what he says is an *ignoratio*. Let us look more closely.

1.1 Material tolerance

It does not suffice to have just *any* connective hold between consecutive claims in a sorites sequence, even one that delivers pairwise equivalence. Thus, take P to be a vague predicate, and its domain to be the set of objects $\{0, 1, 2\}$, such that $P(0)$ and $P(1)$, but not $P(2)$. We gerrymander together the following set of pairs of sentences:

$$
\begin{aligned}
\mathfrak{F} = \{ & \langle P(0), P(0) \rangle \\
& \langle P(1), P(1) \rangle \\
& \langle P(2), P(2) \rangle \\
& \langle P(0), P(1) \rangle \\
& \langle P(1), P(0) \rangle \\
& \langle P(1), P(2) \rangle \\
& \langle P(2), P(1) \rangle \}
\end{aligned}
$$

Then define a connective:

$$A \equiv_{\mathfrak{F}} B := \langle A, B \rangle \in \mathfrak{F}$$

Now we have a relation that is symmetric, reflexive, not transitive (since $P(0) \not\equiv_{\mathfrak{F}} P(2)$), and not detachable:

$$P(1), P(1) \equiv_{\mathfrak{F}} P(2) \nvdash P(2)$$

Can this operator be taken as germane to the sorites paradox? Each of the right pairs of sentences do come out true:

$$P(0) \equiv_{\mathfrak{F}} P(1), P(1) \equiv_{\mathfrak{F}} P(2)$$

but without transitivity or detachment. So the sorites is solved! The analysis has all the virtues of a paraconsistent approach, but (since no mention is made of contradiction) without the cost of inconsistency.

Well, this is bizarre. Unless we do a lot more work, we cannot use $\equiv_{\mathfrak{F}}$ (or any obvious extensions of it into bigger domains of objects) as an approach to the sorites. The problem with $\mathfrak{F}$ is that it is entirely artificial. Something more independently motivated is called for. To take a non-arbitrary example, one much more familiar pairwise-equivalence claim is *material equivalence*: for consecutive claims, A and B, either both A and B, or else neither A nor B, which is (as above) $A \equiv B$, in both LP and classical logic.[3] The material equivalences are at the centre of the LP approach. Rightly so: that consecutive members of the sorites sequence are equally true/false is the key intuition driving the sorites paradox. This is exactly what the material biconditional states. It is the *obvious* statement of tolerance.[4]

If a theory takes the intuition that consecutive members of the sorites sequence are alethically mated to be mistaken, it owes some account of why we are mistaken in this way. Beall offers us no such thing. All he has definitively to say (p. 798) is that we must take the material biconditionals to be false. This is not very illuminating. As he himself says (in a slightly different context), 'unless non-glutty solutions can keep all premisses [of a sorites argument] in an equally natural fashion … the glutty solution should be endorsed' (p. 793). Indeed so.

Further light is cast on the matter if we bear in mind the tentative point made by Beall himself, that his proposal is 'in the vicinity — at least the outskirts — of classical-logic-based epistemicists' (p. 807). As epistemicists freely admit, rejection of $A \equiv B$ is counterintuitive. To compensate, Williamson's epistemic theory (for example) appeals to a margin-of-error principle. Beall's proposal would require a similar supplement. Now, Beall might take assertibility to entail knowledge, and so try to draw on the epistemicist's defence; but this hardly shows that the rejection of $A \equiv B$ is cost-free.[5] After all, the epistemicist's margin-of-error principle is not offered as having anything to do with *tolerance*. No argument is given for thinking that some knowledge-equivalence $\equiv_K$ is a sorites-equivalence connective.[6] Epistemicists do not offer it as such. There is no proposed 'error

[3] Our $\equiv_{\mathfrak{F}}$ above is not material, since $P(1) \equiv_{\mathfrak{F}} P(2)$, but we have stipulated that $P(1)$ is true, and $P(2)$ is not.

[4] Although see Weber and Colyvan 2010 for a generalization of the sorites and a corresponding generalization of tolerance that does not employ conditionals. For present purposes, we set such complications aside.

[5] Such an extension of Beall's proposal would seem to fit naturally were one to endorse the Knowledge Account of Assertion discussed in DeRose 2002. According to this account (see DeRose pp. 179 f): 'The standards for when one is in a position to warrantedly assert that p are the same as those that comprise the truth-conditions for "I know that p".' A weaker claim endorsed in Williamson 2000, Ch. 11, is that one should assert only what one knows.

[6] Define $A \equiv_K B := (KA \supset B)$ & $(KB \supset A)$.

theory' where we mistake $A \equiv_K B$ for $A \equiv B$. Beall thus faces an additional task to that faced by epistemicists: explaining why we should take 'true tolerance' to be expressed as $A \equiv_{Beall} B$. Without some defensive maneuver, otherwise, nothing has been done to diminish the strong intuition that the material equivalence claims in question are simply true. Beall's proposal, in sharing the vices of epistemicism, has clear costs not associated with the LP-based theory.

A main virtue of the LP-based theory, then, is one which the unassertibility-based theory does not share: it allows us to trust the familiar and powerful intuition that consecutive claims in a sorites series are materially equivalent. The LP-based approaches, as Beall recognizes, are almost alone among logical approaches to vagueness in that they do *not* take this intuition to be mistaken.[7] Indeed, they show how it is that the intuition can be accurate without the sorites creating the difficulties it has been supposed to create. Beall's approach, on the other hand, changes the subject: focusing on a claim about assertibility, $A \equiv_{Beall} B$, rather than $A \equiv B$. It is not at all clear why a Beall biconditional, unlike a material biconditional, expresses anything to do with tolerance. Moreover — and crucially — the unassertibility-based approach *must reject* $A \equiv B$, on pain of falling into the sorites paradox.

1.2 Other versions of tolerance

Let us pursue this thought. Beall distinguishes between *extensional* and *intensional* tolerance principles. He claims that the LP-based theorist can allow that extensional (that is, material) tolerance principles hold, but must reject intensional tolerance principles, while the unassertibility-based theorist must reject extensional tolerance principles, but can endorse intensional ones. The positions are symmetric, so that consistency can decide the issue.

Let us pause to suppose the situation is just as Beall claims it to be. Under this supposition, so long as maintaining the truth of extensional tolerance principles is some virtue in a theory of vagueness (as we have argued), it is a virtue the LP theorist enjoys that the unassertibility theorist does not. This alone is enough to falsify Beall's claim that the unassertibility theory has all of the virtues of the LP theory. As we have argued, maintaining the truth of extensional tolerance is such a virtue.

The line between 'extensional' and 'intensional' is notoriously unstable and framework-dependent. Beall wisely does not place much

[7] Almost: see also Cobreros et al. 2012, Zardini 2008.

weight on the difference (see his n. 18), but simply uses the terms as labels, to mark out different tolerance principles one might hold to be important. Extensional versions of tolerance, as Beall uses the term, are the versions that involve the material conditional or biconditional. These are the tolerance principles that LP-based theories are built to perform on, and they are the principles that we have already discussed; the unassertibility-based theory must reject them.

What, then, are intensional tolerance principles? As far as we can see, Beall has two distinct kinds of principle in mind under this heading, and his response depends entirely on conflating them. The first sort of intensional tolerance principle involves $\equiv_{Beall}$; since unassertibility, as Beall understands it, is an intensional notion, these principles count as intensional. Call this sort of tolerance principle *assertibility tolerance*. The second sort involves a detachable conditional — that is, a conditional that validates *modus ponens*. (Perhaps such a conditional could express entailment, or some other strong connection between its antecedent and consequent; the important point here is simply that this sort of conditional is detachable.) Call this sort of tolerance principle *detachable tolerance*. These two sorts are distinct: it is crucial to Beall's unassertibility-based theory that $\equiv_{Beall}$ not be detachable, and so assertibility tolerance principles are themselves not detachable.

Although Beall presents his point as a contrast between two types of tolerance principle — extensional and intensional — he actually considers all three types: extensional, assertibility, and detachable. He claims that his approach can accept intensional tolerance principles, while the LP-based approach cannot. But this is not true of assertibility tolerance claims, since *both* Beall's approach and ours can accept these; and it is not true of detachable tolerance claims, since *neither* approach can accept these. Since these are Beall's only candidates for intensional tolerance principles, we conclude that there are no tolerance principles available to the unassertibility-based approach that are not also available to the LP-based approach. In contrast, there remain important tolerance principles — the extensional ones — available to the LP-based approach that are not available to the unassertibility-based approach.

Moreover, theorists holding to the LP side of the street can, if they want to, accept assertibility tolerance principles as well as the unassertibility-based theorist. Holding to the logic LP does not commit one to any particular theory of assertibility, and so there is no incompatibility at all between holding to an LP-based theory and adding to it Beall's particular development of a theory of assertibility. If endorsing these principles is a good-making feature of a theory of vagueness, it is a good-making feature available to both of the theories in question. On the other hand, the LP theorist has the option, as well, of rejecting these principles for the same reason: no particular theory of assertibility is part of the LP-based approach. So if endorsing these tolerance principles turns out to be a bad-making feature of a theory of vagueness, it is a feature the LP-based theory can avoid, while the unassertibility-based theory cannot.

Assertibility tolerance principles, then, do nothing to compensate for the unassertibility-based approach's rejection of extensional tolerance principles: if the assertibility tolerance principles should be accepted, everyone can accept them. On the other hand, if they should not be accepted, the unassertibility-based approach is doubly worse off: it is committed to these principles, while an LP-based approach need not be.

1.3 Detachable tolerance

What, then, of detachable tolerance principles? Together with the assumption that logical consequence is transitive,[8] detachable tolerance principles run a theory straight into the business end of the sorites paradox. These principles, it follows, are not acceptable to an LP-based theorist — but neither are they available to an unassertibility-based theorist.

In fact, this is the very reason that *extensional* tolerance principles are unavailable to the unassertibility-based approach: that approach must see the extensional principles as detachable, and so must reject them. An LP-based approach, on the other hand, has room to see the extensional principles as non-detachable, allowing for them to be accepted.

If accepting detachable tolerance principles is a good-making feature of a theory of vagueness, it is a good-making feature that the unassertibility-based theory, like the LP-based theory, must lack. If it is a bad-making feature, then it is a bad-making feature they both must lack. In neither case does it do anything to compensate for the unassertibility-based approach's rejection of extensional tolerance principles.

Are there any other detachable tolerance principles around which could make serious trouble for the glutty theorist? An LP-based

[8] This is an assumption that both LP-based theories and unassertibility-based theories share, but it is not mandatory; see, again, Cobreros et al. 2012 and Zardini 2008.

approach holds that any expression of some sort of tolerance condition which employs a detachable conditional is not true. In many cases, this is entirely obvious. If P is a vague predicate, and x and x' are consecutive members of a sorites sequence, then $Px \rightarrow Px'$ is clearly untrue if $\rightarrow$ expresses logical entailment. Beall raises the question of 'full-dressed' tolerance principles:

$$Px \,\&\, Rxx' \rightarrow Px'$$

where R is a condition such as 'are indiscriminable'. He claims that '[o]nce we fully dress … [the conditionals], the considerations that target glutty philosophers mount against the plausibility of intensional versions of tolerance — namely, too strong to be plausible — seem to disappear' (p. 805).

As we have just noted, this is no argument for his approach, for he is committed to denying such principles no less than LP-based theorists are. But the claim is not true anyway. Such principles are really no more plausible than the 'half-dressed' ones. There is still no logical entailment between the antecedent and the consequent of the 'full-dressed' conditional. Indeed, whatever considerations one might bring against the 'half-dressed' versions, one would appear to be able to bring analogous considerations against the 'full-dressed' one. The intuitive tolerance principle is exactly that Px and Px' are either both true or both false — just as the material biconditional has it. There are no tighter connections. Bring the dressing in, if you like:

$$Rxx' \rightarrow (Px \equiv Px')$$

It does not change matters.

So, after considering the sorts of tolerance principle Beall mentions, we can gather up this line of argument. The original objection stands: Beall's claim that the unassertibility-based theory has all the virtues of the LP-based theory is false. An LP-based approach can accept extensional tolerance principles, and Beall's unassertibility-based approach cannot; this is a key virtue of LP-based approaches, and it is not shared by the proffered replacement. Moreover, none of the other ('intensional') tolerance principles Beall mentions make up for this shortfall in the unassertibility-based theory, since they are either assertibility tolerance principles, which *either* approach can accept without trouble, or else detachable tolerance principles, which *neither* approach can accept without trouble. In sum, then, his approach and ours are not symmetrically paired when it comes to tolerance. And the asymmetry around tolerance principles is entirely to the advantage of the paraconsistent approach.

2. The virtues of contradiction

There are two other objections to consider concerning the explanatory resources of an LP-based approach. The notion of *explanation* is perhaps even more unstable than that of being *intensional*, but it is also more familiar and forms an important pillar in the paraconsistent theory. That a theory of vagueness is required to do some explaining should be obvious, and is borne out, for example, by reviewing the debate around epistemicism, which is often charged with positing *inexplicably* sharp cut-offs.[9]

The paraconsistent approach, we claim, is preferable to Beall's approach since it has explanatory power which Beall's lacks. Indeed, on our approach, gluts are not a *cost* to be mitigated and, if possible, avoided: they provide substantial explanatory power. Beall contests this claim. Let us examine.

2.1 Inclosure paradoxes

The notion of explanation arises in the first objection Beall considers (Sects 4.1, 4.2). We claim that the sorites paradox is an inclosure paradox, of a kind with the paradoxes of self-reference.[10] This association gives a strong *reason* (at least as strong a reason as in the case of the Liar) to take a paraconsistent approach, and subsequently to theorize $\equiv$ using LP. At the same time, recall that the inclosure schema is essentially due to Russell, himself no dialetheist; so an inclosure analysis of a paradox is independent of, and hardly mandates, a glutty approach (Weber 2010a). An inclosure analysis identifies a principled and unifying relationship between paradoxes. In so far as it aspires to explain all the usual paradoxes of self-reference and sorites paradoxes, inclosure very nicely accords with, for example, Kitcher's idea of

[9] For example, in Smith 2008, Ch. 4.

[10] The observation that sorites paradoxes may be seen as inclosure paradoxes was first made by Priest in the discussion following the presentation of Colyvan (2009) (which draws other similarities between sorites paradoxes and the paradoxes of self-reference) at the 2007 meeting of the Australasian Association of Philosophy. Chase (MS) also observed that continuous sorites arguments satisfy the inclosure conditions. The connection between sorites paradoxes and the inclosure schema, amongst other things, was discussed at a working group on sorites paradoxes at the University of Sydney later in 2007, where most of us were present. See chapters by Hyde and Priest in Berto et al. 2012.

unification (1981). The inclosure first gives a diagnosis, and then recommends a principled response.

Beall's proposed replacement account does nothing of this kind. His theory neither explains nor rests upon any detailed exposition of the phenomenon of vagueness. In our glut-based theories, this is the role that contradictions arising out of the inclosure schema play. Inconsistency is an emergent property of an inclosure structure, and is *itself* the explanation for the entire sorites phenomenon. The structure of an inclosure explains *why* contradictions occur at the vague penumbra (Priest 2002, Sects 17.2, 17.6; Priest 2010; Colyvan 2009). As Priest (2002, p. 136) puts it:

> Once one understands *how* it is that a diagonaliser manages to operate on a totality of objects of a certain kind to produce a novel object of the same kind, it becomes clear *why* a contradiction occurs at the limit.

Beall's account provides no similar explanation of the murky goings-on in the middle of a sorites progression.

Beall replies that the inclosure schema cannot carry the explanatory weight required, simply because it does not characterize the relevant class of paradoxes. In particular, he claims that the Curry paradox fits the schema, though the solution to this paradox is not a dialetheic one; Curry's paradox must be solved some other way. This is an interesting and important point.

The inclosure schema concerns monadic predicates, φ, ψ, and a monadic function, δ, which, prima facie, satisfy certain conditions (Priest 2002, p. 156; 2010).[11] Curry's paradox is said to fit the inclosure schema where these are as follows (and T is the truth predicate):

$\varphi(y)$:='Ty'

$\psi(x)$:='x is definable'

$\delta(x)$:=s

where s is a sentence of the form $\langle s \in \dot{x} \rightarrow \bot\rangle$, and $\dot{x}$ is a name for x. Let $\Omega = \{y : y \text{ is true}\}$.

The function $\delta(x)$ is clearly defined when $\psi(x)$ holds. The set Ω exists since it is a set of sentences of some (countable) language, and it

[11] Beall says that the Barber paradox is not an inclosure paradox, since there is no prima facie reason to suppose the conditions are met. We concur. He also says the same of the Secretary Club (pp. 7–8). Here we demur. Though (to our knowledge) there is no such club, there *could* be. There is nothing in principle impossible about a group of people duly authorizing a constitution of the kind involved.

is obvious that $\psi(\Omega)$. Take an x such that $x \subseteq \Omega$ and $\psi(x)$. Suppose that $\delta(x) = s$ and $s \in \dot{x}$. Then, since $x \subseteq \Omega$, it follows that s is true, and so $s \in \dot{x} \to \bot$, which gives $\bot$. Discharging the supposition, $s \in \dot{x} \to \bot$; that is, Ts; that is $s \in \Omega$ (closure). But since $s \in \dot{x} \to \bot$ is true, if $s \in \dot{x}$ then $\bot \in \Omega$, which it is not. Hence, by contraposition, $s \notin \dot{x}$ (transcendence). The limit contradiction arises when x is Ω. Then $s \in \Omega$ and $s \notin \Omega$, where $\delta(\Omega) = s$.

So Curry's paradox fits the schema? No; this is not Curry's paradox![12] The problem posed by Curry-style reasoning is that it allows us to establish an *arbitrary* sentence, not just $\bot$. If A is any sentence, and s is the sentence $Ts \to A$, the Curry reasoning allows us to establish A. This is problematic whether A is true or false. There is obviously something crazy about supposing that we can establish (the truth) that Canberra is the capital of Australia in this way, for example. And once one sees this, it is clear that the paradox does not fit the inclosure schema. Specifically, if we replace $\bot$ with a true A in the above reasoning, the argument to transcendence fails, since we can no longer appeal to contrapostition. Of course, individual instances of the Curry reasoning, say when A is $\bot$, may satisfy the inclosure conditions.[13] But that does not show that the Curry paradox itself is an inclosure paradox — any more than the fact that there are instances of cubic equations, $ax^3 + bx^2 + cx + d = 0$, which are quadratic (when $a = 0$) shows that cubic equations are quadratic. And it is bad form to take special cases to deliver general conclusions. Hard cases make bad law.

Beall is attempting to appeal to the *principle of uniform solution*: same kind of paradox, same kind of solution (Priest 2002, Sects 11.5, 17.6). What is at issue, then, is the question of whether the Curry paradox is an inclosure paradox. All the instances of the Curry paradox — whatever the A is — are clearly of the same kind. And it is not an inclosure paradox, since instances of it do not fit the schema. By the uniform solution principle, the Curry paradoxes all require the same treatment, but not (necessarily) the same as that of the Liar, the sorities, etc. As Priest 2002 notes (p. 169), Curry's paradox has nothing essentially to do with negation: it would arise even if the language

[12] Of course Curry's paradox is a paradox that employs self-reference, but there are many paradoxes that employ self-reference which are clearly of quite a different kind from the Liar, etc., such as versions of the Surprise Exam paradox. See Priest 2002, Sect. 17.2.

[13] The inclosure conditions are only prima facie true of the Curry sentence when A is $\bot$. They are not true if given a suitable paraconsistent logic, e.g. one in which the logical rule of contraction fails. In particular, then, they do not produce a proof of $\bot$.

were entirely positive. It therefore has nothing to do with contradictions that occur at various boundaries (limits), which is what inclosures are all about.[14]

2.2 Cut-offs

The presence of contradiction at the limit of an inclosure leads to the issue of cut-off points. The paraconsistent approach explains why soritical progressions have a cut-off point, and therefore do not overextend into absurd conclusions like 'everything is red'. They do this exactly because there is a sharp cut-off point where these progressions hit an inconsistency. Of course, it is notoriously hard to locate such a point: anywhere in the penumbra seems just as good. The paraconsistent account explains this too. There is no unique cut-off point: there are many (as Beall nicely demonstrates in his appendix).

Beall demurs: every cut-off point is itself a unique cut-off point. So we do not avoid the problem; we just multiply it. But as Weber (2010b, p. 1038) explains, this just fails to take the inconsistency seriously. Why is a *unique* cut-off point problematic? Because it is hard to locate it. And the reason that it is just as hard to locate it is the fact that there are many of them (again, see Beall's appendix). The situation is, in fact, *exactly* the same as with cut-offs to begin with. Adding the word 'unique' changes nothing. We are in the realm here of 'higher-order vagueness', on which see Priest 2010. If carried to its full expression, a dialetheic explanation is that, in so far as all cut-offs *are* unique, then the existence of a cut-off is itself a glut: there is and is not a cut-off point (Weber 2010).

What is driving Beall's objection is, in fact, the thought that if there is a cut-off at all then there must be a *real* (first) cut-off point. And 'real' here means *consistently specifiable*. We agree that if there are cut-off points then they are real, but do not equate reality with consistency; that is the whole idea of the approach. Appeals to consistency of this sort are inappropriate in dialetheic contexts — as Beall has himself argued (Beall et al. 2011).

[14] Since the only connective employed is the conditional, it is reasonable to suppose that some assumption concerning the behaviour of this is wrong. Contraction ($A \to (A \to B) \models A \to B$) is the most obvious candidate (especially given the fact that there are natural accounts of the conditional which show this to be an invalid principle, such as contraction-free relevant logics). Alternately, some structural rule governing the problematic derivation may be to blame; structural contraction (distinct from contraction!) and transitivity seem to be the potential culprits here. See Beall and Murzi 2013 and Ripley 2013 for discussion of noncontractive and nontransitive approaches, respectively.

So we stand by our claims that the boundary of a soritical progression is an inclosure, and that the emergent contradiction goes on to explain facts about cut-off points. Beall's account has no such explanatory power. One cannot simply introduce an operator, detachable or otherwise, and expect this to deliver a theory of vagueness. We already know that not everyone is bald, and that the existence of a 'last' bald man is puzzling; the project is to say *why*. Inconsistency is our answer, and it is an answer that does a lot of satisfying work.

3. Assertibility without gluts?

Let us close with a final observation. Beall assumes (as does almost everyone else) that, all else being equal, a consistent theory is better than an inconsistent theory. Given his other views (Beall 2009), this is more significant than usual, and 'all else being equal' carries a lot of weight; for Beall accepts that some inconsistent theories are, all things considered, better than consistent ones. Thus, Beall's main supposed selling point for his approach is that it has all the advantages of a paraconsistent approach without the costs. We have spent much time contesting the first part of the claim; but the second is also moot.

In the postscript to his paper, Beall points out the close similarities between an assertibilty operator, in the sense that he uses it, and a knowledge operator. But knowledge operators are notoriously implicated in dialetheias, in the form of knower paradox (Priest 2002, Sect. 10.2):

It is not the case that this sentence is known to be true

The paradox in terms of assertibility goes as follows. Consider a sentence, *a*, of the form '*a* is not assertable'. Suppose that *a* is assertable. Then, *a* is true (p. 794). So *a* is not assertable. Hence, by *reductio*, *a* is not assertable. But we have just established this. Hence, we can assert it. So *a* is assertable — as well. It is precisely not the case, then, that one may purchase freedom from contradiction by invoking the notion of assertibility.

Beall would probably reply that though contradiction about knowledge might have to be accepted in the case of self-reference, it does not follow that it should be accepted in the case of vagueness: that is quite a different topic of reasoning. If both the knower paradox and the sorites paradox are inclosure contradictions, as we have argued, this move is undercut. But we note that even Beall seems to be prepared to countenance the possibility of dialetheias concerning knowledge in realms that do not involve self-reference, where such a view has appropriate explanatory power, for instance, in the case of Fitch's paradox (Beall 2000).[15]

4. Conclusion

The purpose of a theory of vagueness is to provide a coherent story about the ubiquitous, innocuous, and intransigent facts about sorites progressions. The LP-based solutions do this — by analysing the structure of sorites paradoxes, and by giving formal details according to which all the right intuitions are captured — without slipping into absurdity. 'Tolerance without gluts' lacks both of these virtues. Not only can it not explain the apparent truth of the relevant material conditionals; it lacks the *general* explanatory power concerning the phenomenon of vagueness which the LP account possesses. Beall says that '[t]he major benefit of a glut-based approach is maintaining the truth of all sorites premisses while none the less avoiding, in a principled fashion, the absurdity of the sorites conclusion(s)' (p. 791). This it does: an LP-based approach remains almost alone in maintaining intuitive forms of sorites premisses while averting absurdity in a principled way. But Beall's dictum sells the approach short. By attempting to omit the most important feature of the paraconsistent approach — justified contradictions and their explanatory power — Beall has helped highlight exactly how important gluts can be.[16]

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[15] Beall also once toyed with the idea of inconsistent theories of vagueness (Beall and Colyvan 2001), although more recently he has distanced himself from this view.

[16] Thanks to Jc Beall. Research for this paper was funded in part by ARC Discovery Project 1094962.

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