Mind, Vol. 109; April 2000, pp. 390–394

The Applicability of Mathematics as a Philosophical Problem, by Mark Steiner. Cambridge MA: Harvard University Press, 1998. Pp. viii + 215. USD 39.95

It's difficult not to be impressed by the remarkable success mathematics has enjoyed in its various applications in physical science. Indeed, this success borders on the mysterious. As Eugene Wigner suggests in a much-cited passage from 'The Unreasonable Effectiveness of Mathematics in the Natural Sciences' (*Communications on Pure and Applied Mathematics*, 13, pp. 1–14):

> The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. (p. 14)

Mark Steiner takes up this puzzle in *The Applicability of Mathematics as a Philosophical Problem*:

> [H]ow does the mathematiciancloser to the artist than to the explorerby turning away from nature, arrive at its most appropriate descriptions? (p. 47)

In this book, Steiner presents a detailed discussion of the philosophical issues surrounding the applicability of mathematics. He argues ultimately that the success of mathematics in applications (especially in modern theories such as quantum mechanics) presents an apparent violation of a widely-held doctrine that we might call 'anti-anthropocentrism' (or as Steiner prefers to call it, 'naturalism'). The basic idea is that our choice of mathematical concepts, and indeed what we count as mathematics, is driven by aesthetic considerations and convenience (which are irredeemably anthropocentric) and yet these mathematical concepts play an important role in both the discovery and articulation of our best physical theories. In short, the universe appears to be "user friendly". The book is clear, despite its often technical subject matter, and the main theses are well argued for. It's packed with interesting examples from physics—particularly from quantum mechanics. Although some of these examples may be familiar to philosophers of physics, I found Steiner's careful attention to the role mathematics played in the development of the theories in question was interesting both from the point of view of a defence of the main thesis of the book, but also in helping one see these theories from a different and somewhat more abstract point of view.

As Steiner points out, the problem of accounting for the applicability of mathematics has not received the philosophical attention that you might expect. One reason for this is that it has not always been clear what "the problem" of the applicability of mathematics is. A significant contribution

Steiner makes in this book is in clarifying this issue. He begins by arguing that there are in fact many problems associated with the applicability of mathematics—it all depends on how you take mathematics to be applied. For example, there's the problem of explaining the *semantic* applicability of mathematics. This is the problem of explaining the validity of mathematical reasoning in both pure and applied contexts—to explain, for instance, why the truth of (i) there are 6 small planets, (ii) there are 3 large planets and, (iii) 6+3=9, implies that there are 9 planets. (In (i) and (ii) '6' and '3' act as names of predicates and yet in (iii) '6' and '3' act (apparently) as names of objects.) We thus require an interpretation of the mathematical vocabulary that is constant across pure and applied contexts. The solution to this problem, Steiner argues (following Michael Dummett in *Frege: Philosophy of Mathematics* (Cambridge MA: Harvard University Press, 1991)), was solved long ago by Frege and yet it is often confused with other problems associated with the applicability of mathematics. Steiner is very careful to disentangle the various problems and he presents a version of Wigner's thesis that is both more precise than Wigner's and is immune from some obvious objections.

Steiner's version of Wigner's puzzle focuses on the role mathematics plays in discovery. Whereas Wigner was impressed by the role mathematical concepts played in our descriptions of the world, Steiner is impressed by the roles mathematics has played in the very discovery of those descriptions. Moreover, Steiner argues that "what has been astonishingly successful was a grand strategy, not an isolated act, and what succeeded was the use of the entire structure of mathematical concepts, not this or that concept" (p. 9). That mathematics should be successful in this way is puzzling because it smacks of anthropocentrism.

> [R]elying on mathematics in guessing the laws of nature is relying on human standards of beauty and convenience. So this is an anthropocentric policy; nevertheless, physicists pursued it with great success. (p. 7)

The point is not that mathematical deduction and proof is anthropocentric; rather, it's that the choice of concepts studied and the choice of notation employed is anthropocentric. The concepts studied are (in part) those that mathematicians find aesthetically pleasing and the notation employed is (largely) a matter of convenience.

Steiner goes on to describe two quite different, but equally puzzling, ways in which mathematics has facilitated the discovery of physical theories. The first way Steiner calls 'Pythagorean analogy'. This is where an analogy is drawn between two descriptions of physical systems and the analogy is based purely on the classification schemes of mathematics. That is, the analogy has no known non-mathematical basis. Steiner offers the following instance of this strategy:

> Equation E has been derived under assumptions A. The equation has solutions for which A are no longer valid; but *just because they are solutions of E*, one looks for them in nature. (p. 76, Steiner's italics)

Such a strategy is 'Pythagorean' because the solutions to the equation derived under its assumptions A and those solutions derived when A are no longer valid are physically disanalogous; only the equation ties them together. Steiner presents many fascinating examples of this kind of strategy from the history of modern physics. One rather well-known example is Dirac's refusal to discard apparently non-physical, negative-energy solutions to the equation that now bears his name, and how this led to the discovery of anti-matter.

The second way in which mathematics facilitates discovery Steiner calls 'formalist analogy'. Here mathematical *notation* is driving the discovery. In particular, Steiner has in mind cases where there was no interpretation of the notation in question.

> [T]he equations [the physicists] guessed simply looked like the equations they already had. In such cases, scientists were studying their own representational system—i.e., themselves—more than nature. (p. 7)

Again such apparently anthropocentic tactics proved to be remarkably successful in modern science—particularly in quantum mechanics. The final chapter of Steiner's book is devoted to discussing a number of examples of the "quantization" strategy used in developing modern quantum mechanics and showing how these were instances of the formalist strategy. Here physicists falsely assumed that the system in question obeyed classical laws. (Indeed, in the cases Steiner considers, they *knew* that this assumption was false.) They then performed syntactic manipulation of the classical description in order to obtain what they hoped (and what all too often) was an accurate quantum description of the same system. Steiner suggests that "discoveries made this way relied on symbolic manipulations that border on the magical" (p. 136). Perhaps the most astonishing example of this formalist strategy in quantum mechanics is seen in Yang's and Mills' introduction of gauge field theories. Yang and Mills, driven by group-theoretic considerations, quantized a fictitious classical equation (itself arrived at by analogy with classical field equations such as Maxwell's equations) arriving at a quantum field equation. At every turn their reasoning was formalist and at the end of they day there was little reason to believe that their quantum field equation was even consistent! Yet the remarkable success of gauge field theories speak for themselves.

I now turn to some questions that Steiner's book raises but which are not dealt with in any detail. First, there are questions about the role of aesthetic considerations in science generally. It is well known that empirical evidence underdetermines theories, so in order to obtain a unique theory scientists often appeal to aesthetic (or at least non-empirical) considerations such as simplicity and elegance. It would have been good to hear Steiner's thoughts on the relationship between his thesis (that the use of mathematics in physical theories is at odds with anti-anthropocentrism) and the criticism of the appeal to aesthetics made by scientific anti-realists (and others). The second question is that if Steiner is right and the universe *is* user friendly (that is, human consciousness holds a privileged place in the universe), what are the consequences of this? Although Steiner falls short of drawing any theistic conclusions, such conclusions are beckoning! (Indeed, his thesis appears to be closely related to design arguments.) Finally, I was left wondering about whether the anti-realism/realism debate in mathematics has any bearing on Steiner's thesis. I have in mind here the fact that the usual presentation of Wigner's puzzle at least hints at an anti-realist philosophy of mathematics. (For example, see the passage from Steiner I quoted earlier about the mathematician being closer to the artist than the explorer.) Although Steiner intends his discussion to be independent of realist/anti-realist issues in the philosophy of mathematics, it is not always clear that this is so.

The Applicability of Mathematics as a Philosophical Problem is a valuable addition to the philosophy of science and philosophy of mathematics literature. It presents a rigorous and detailed presentation of a puzzle that I believe is crying out for attention. Whether or not you agree with Steiner's disturbing conclusion that the "miracle" of the applicability of mathematics suggests that we live in a user-friendly universe, there is no doubt about the importance of this book. The problems associated with the applicability of mathematics can no longer be ignored.

School of Philosophy
University of Tasmania
GPO Box 252-41
Hobart, Tasmania, 7001
Australia

MARK COLYVAN