Ranking in Threatened Species Classification: Reply to Todd
Author(s): Mark Colyvan
Source: *Conservation Biology*, Vol. 15, No. 6 (Dec., 2001), pp. 1814-1816
Published by: Blackwell Publishing for Society for Conservation Biology
Stable URL: http://www.jstor.org/stable/3061284
Accessed: 29/01/2009 20:26

Comments

Ranking in Threatened Species Classification: Reply to Todd

MARK COLYVAN

School of Philosophy, University of Tasmania, GPO Box 252-41, Hobart, Tasmania, 7005, Australia, email mark.colyvan@utas.edu.au

Introduction

There are many reasons to be dissatisfied with Todd's (2000) reply to Regan's and my recent Comment (Regan & Colyvan 2000), not least of which is that his reply does not advance the debate. Moreover, he has made the method of Todd and Burgman (1998) even more unclear. I will not dwell on the various shortcomings of Todd's reply—they should be apparent to any careful reader. Instead, I focus on the most important matter raised by Todd: the issue of the appropriate intersection and union operators for the method and how this affects interpretation of the results.

I should first point out that this issue is not merely a minor, technical quibble without practical import. The matters Regan and I (2000) raised in our earlier discussion and the point I discuss here are of the utmost importance to conservation biologists interested in using fuzzy methods in their work. For example, if the issue I raise in the present discussion is ignored and the method of Todd and Burgman is employed, misclassification of species with respect to conservation status will be likely because a crucial detail of the mathematical formalism is left unspecified. Todd claims that the details of the fuzzy intersection and union operators used in their method is not important because the rankings of species under this method, no matter how the missing details are filled out, remain the same. I will demonstrate here that this is not true. Thus, Todd cannot guarantee that the method will produce consistent results; the results will depend on how the mathematical formalism is specified.

The Conjecture and the Counter Example

When discussing the various fuzzy intersections and union operators that might be employed in a certain conservation management application, Todd and Burgman do not commit themselves to any particular operators. They first discuss the standard minimum and maximum operators and then present the algebraic product and sum operators. They suggest that the latter have "some intuitive appeal for risk assessment" (Todd & Burgman 1998:972). In our Comment (Regan & Colyvan 2000) we assumed that the algebraic product and sum operators were the preferred options. Todd (2000) assures us that we were mistaken in this assumption. We stand corrected. But instead of telling us which operators *are* endorsed, Todd repeats an unsubstantiated claim made in the earlier paper that it does not matter which operators are chosen: "Regan and Colyvan failed to acknowledge that it does not matter which operator is used; the rank order of fuzzy sets (largest degree of membership to smallest) remains the same" (Todd 2000:1200).

What Todd has in mind here is made clearer in some earlier remarks on this matter (which, again, I quote so that there can be no confusion about what is intended): "The fuzzy subset 1a ∩ 3a has the highest degree of membership under both operators [min and algebraic product] (Table 2) [of Todd & Burgman 1998]. This is a general property of the intersection and union of fuzzy sets, in which the degree of membership under different operators varies. Regardless of the operator applied, the combination that produces the largest degree of membership is the same across operators" (Todd & Burgman 1998:972).

The suggestion is that if some element x in the universe of discourse Ω has membership $\mu_A(x)$, $\mu_B(x)$, $\mu_C(x)$, and $\mu_D(x)$ in four fuzzy subsets A, B, C, and D of Ω, respectively, and if we rank the various intersections of these four sets according to x's membership in them, this ranking does not depend on the t-norm intersection operator used. (A similar result is supposed to hold for t-conorm union operators.) That is, if for example $\mu_{A \cap B}(x) < \mu_{C \cap D}(x)$ or $\mu_{A \cap B}(x) = \mu_{C \cap D}(x)$, then the respective relation holds for *all* choices of t-norm intersection operators (and likewise for unions under the various t-conorm union operators). Call these the t-norm and t-conorm ordering conjectures, respectively. In particular, Todd (2000) claims that the rank order of the membership values of x

Paper submitted October 2, 2000; manuscript accepted July 23, 2001.

in $A \cap B$ and in $C \cap D$ do not depend on whether one uses the min-intersection operator or the algebraic-product intersection operator.

The reason we failed to acknowledge the t-norm ordering conjecture is that it is straightforwardly false. Moreover, the falsity of this conjecture is apparent from Table 2 of Todd and Burgman (1998): 1b ∩ 3a and 1b ∩ 3b are ranked the same according to the min-intersection operator, and yet 1b ∩ 3a is ranked higher than 1b ∩ 3b according to the algebraic-product operator. Rather than "rank order remains the same," perhaps Todd really means the order is not reversed. But this too is false, as the following counter-example illustrates. Let $\mu_A(x) = 1/2$, $\mu_B(x) = 1/14$, $\mu_C(x) = 1/15$, and $\mu_D(x) = 3/4$. Under the algebraic-product operator,

$$\begin{aligned} \mu_{A \cap B}(x) &= 1/2 \times 1/14 = 1/28 < \\ \mu_{C \cap D}(x) &= 1/15 \times 3/4 = 1/20. \end{aligned} \quad (1)$$

But under the min-intersection operator the order is reversed:

$$\begin{aligned} \mu_{A \cap B}(x) &= \min\{1/2, 1/14\} = 1/14 > \\ \mu_{C \cap D}(x) &= \min\{1/15, 3/4\} = 1/15. \end{aligned} \quad (2)$$

Similar clear-cut counter-examples are readily produced for the t-conorm ordering conjecture. (For example, let $\mu_A(x) = 3/8$, $\mu_B(x) = 1/2$, $\mu_C(x) = 5/8$, $\mu_D(x) = 0$, and consider $A \cup B$ and $C \cup D$ under the algebraic-sum and max-union operators.) The bottom line is that ordering depends crucially on the operators employed—the ordering conjectures are false.

Discussion

It is clear that Todd must acknowledge which operators he endorses. Just as important, he needs to justify the choice. This obligation also serves to highlight our earlier point (Regan & Colyvan 2000) about the need to provide an interpretation of the results. At present, Todd has simply told us that a species is related to various sets of species by various numbers. What do these numbers represent? And what is the significance, if any, of an ordering based on them? This is an important issue because mathematics can give us information about the world only if a suitable interpretation of the mathematics involved is provided.

It seems that there are only two interpretations available for the method, but neither of these is acceptable. The first is that the numbers in columns 2 and 3 of Table 2 of Todd and Burgman (1998) are degrees of membership of some species in the sets in question. But, as we've already pointed out (Regan & Colyvan 2000) and as Todd explicitly acknowledges, the sets in question (Millsap categories) are sharp. They simply do not permit degrees of membership, so this interpretation is not viable.

The other possible interpretation of these numbers is that they are probabilities, but this too is problematic. This interpretation is plausible only if the algebraic product and sum operators are endorsed (because these are the fuzzy set-theoretic analogues of probabilistic intersection and union, if dependencies are ignored). But if these operators are endorsed, then our earlier objection (Regan & Colyvan 2000) stands: the whole excursion into fuzzy set theory is unnecessary. On the other hand, if different operators are employed, the option of interpreting the numbers as probabilities is not viable unless the resulting algebra obeys the Kolmogorov axioms. It is easily shown, for example, that fuzzy set theory, with the usual minimum and maximum operators, is not such an algebra. This also means that the results cannot be interpreted as subjective probabilities (or degrees of belief) unless the resulting algebra satisfies the Kolmogorov axioms, because it is also well known that subjective probabilities satisfy the Kolmogorov axioms (Resnik 1987). (See Regan et al. [2000] and Colyvan [2001] for discussion of the different sorts of uncertainty that underwrite the differences between probabilistic and fuzzy methods.)

Finally, it is important to note the differences between the proposal of Todd and Burgman (1998) and the proposal of Akçakaya et al. (2000) because there is the potential for confusion. Akçakaya et al. (2000) use fuzzy numbers and intervals to bound the uncertainty about parameters, rather than specifying the probability that a species is an element of a particular set, as Todd and Burgman do. Akçakaya et al. also recognize the importance of interpreting the mathematics used and provide the details of the algebra.

Until Todd addresses the issues we have raised here and earlier (Regan & Colyvan 2000), a serious question hangs over the method. Indeed, without a recommendation for the appropriate operators and without an interpretation of the results, it is somewhat misleading to call it a "method" at all. In conclusion, I agree with Todd that the issue of whether probabilities can be used as fuzzy-membership functions is a point of contention. It should be clear to all, however, that this is not the only point of contention.

Acknowledgments

I thank H. Regan for ongoing discussion on this and related topics. I also acknowledge useful exchanges with P. Wade and R. Yager.

Literature Cited

Akçakaya, H. R., S. Ferson, M. A. Burgman, D. Keith, G. Mace, and C. R. Todd. 2000. Making consistent IUCN classifications under uncertainty. Conservation Biology 14:1001-1013.

Colyvan, M. 2002. Is probability the only coherent approach to uncertainty? Risk Analysis: in press.

Regan, H. M., and M. Colyvan. 2000. Fuzzy sets and threatened species classification. Conservation Biology **14**:1197-1199.

Regan, H. M., M. Colyvan, and M. A. Burgman. 2000. A proposal for fuzzy IUCN categories and criteria. Biological Conservation **92**:101-108.

Resnik, M. D. 1987. Choices: an introduction to decision theory. University of Minnesota Press, Minneapolis.

Todd, C. R. 2000. Perspectives on the definition of fuzzy sets: a reply to Regan and Colyvan. Conservation Biology **14**:1200-1201.

Todd, C. R., and M. A. Burgman. 1998. The assessment of threat and conservation priorities under realistic levels of uncertainty and reliability. Conservation Biology **12**:966-974.