# Mating, Dating, and Mathematics: It's All in the Game

*Mark Colyvan*

Why do people stay together in monogamous relationships? Love? Fear? Habit? Ethics? Integrity? Desperation? In this essay I will consider a rather surprising answer that comes from mathematics. It turns out that cooperative behaviour, such as mutually-faithful marriages, can be given a firm basis in a mathematical theory known as *game theory*. I will suggest that faithfulness in relationships is fully accounted for by narrow self interest in the appropriate game theory setting. This is a surprising answer because faithful behaviour is usually thought to involve love, ethics, and caring about the well being of your partner. It seems that the game-theory account of faithfulness has no need for such romantic notions. I will consider the philosophical upshot of the game-theoretic answer and see if it really does deliver what is required. Does the game-theoretic answer miss what is important about faithful relationships or does it help us get to the heart of the matter? Before we start looking at lasting, faithful relationships, though, let's get a feel for how mathematics might be employed to help in matters of the heart. Let's first consider how mathematics might shed light on dating to find a suitable partner.

## A Lover's Question

Consider the question of how many people you should date before you commit to a more permanent relationship such as marriage. Marrying the first person you date is, as a general strategy, a bad idea. After all, there's very likely to be someone better out there, but by marrying too early you're cutting off such opportunities. But at the other extreme, always leaving your options open by endlessly dating and continually looking for someone better is not a good strategy either. It would seem that somewhere between marrying your first high-school crush and dating forever lies the ideal strategy. Finding this ideal strategy is an optimisation problem and, believe it or not, is particularly amenable to mathematical treatment. In fact, if we add a couple of constraints to the problem, we have the classic mathematical problem known as *the secretary problem*.

The mathematical version of the problem is presented as one of finding the best secretary (which is just a thin disguise for finding the best mate) by interviewing (i.e. dating) a number of applicants. In the standard formulation, you have a finite and known number of applicants and you must interview these $n$ candidates sequentially. Most importantly, you must decide whether to accept or reject each applicant immediately after interviewing him or her; you cannot call back a previously-interviewed applicant. This makes little sense in the job-search context but is very natural in the dating context: typically, boyfriends and girlfriends do not take kindly to being passed over for someone else and are not usually open to the possibility of a recall. The question, then, is how many of the $n$ possible candidates should you interview before making an appointment? Or in the dating version of the problem, the question is: how many people should you date before you marry?

It can be shown mathematically that the optimal strategy, for a large applicant pool (i.e. when $n$ is large) is to pass over the first $n/e$ (where $e$ is the transcendental number from elementary calculus—the base of the natural logarithm, approximately 2.718) applicants and accept the next applicant who's better than all those previously seen. This gives a probability of finding the best secretary (mate) at $1/e$ or approximately 0.37. For example, suppose that there are one hundred eligible partners in your village, tribe, or social network; this strategy advises you to sample the population by dating the first 37, then choose the first after that who's better than all who came before. Of course, you might be unlucky in a number of ways. For example, the perfect mate might be in the first 37 and get passed over during the sampling phase. In this case, you continue dating the rest but find no one suitable and grow old alone, dreaming of what might have been. Another way you might be unlucky is if you have a run of really weak candidates in the first 37. If the next few are also weak but there's one who's better than the first 37, you commit to that one and find yourself in a sub-optimal marriage. But the mathematics shows that even though things can go wrong in these ways, the strategy outlined here is still the best you can do. The news gets worse though: even if you stringently following this best strategy, you still only have a bit better than a one in three chance of finding your best mate.[1]

This problem and its mathematical treatment are instructive in a number of ways. Here I want to draw attention to the various idealisations and assumptions of this way of setting things up. Notice that we started with a more general problem of how many people you should date before you marry, but in the mathematical treatment, we stipulate that the population of eligible partners is fixed and known. It's interesting that the size of this population does not change the strategy or your chances of finding your perfect partner—the strategy is as I just described and, so long as the population is large, the probability of success remains at 0.37. The size of the population just affects the number of people in the initial sample. But, still, stipulating that the population is fixed is an idealisation. Most pools of eligible partners are not fixed in this way—we meet new people, and others who were previously in relationships later become available, while others who were previously available enter new relationships and become unavailable. In reality, the population of eligible candidates is not fixed but is open-ended and in flux.

The mathematical treatment also assumes that the aim is to marry the best candidate. This, in turn, has two further assumptions. First, it assumes that it is in fact possible to rank candidates in the required way and that you will be able to arrive at this ranking via one date with each. We can have ties between candidates but we are not permitted to have cases where we cannot compare candidates. The mathematical treatment also assumes that we're after the *best* candidate, and anything less than this is a failure. For instance, if you have more modest goals and are only interested in finding someone who'll meet a minimum standard, you need to set things up in a completely different way—it then becomes a satisficing problem and is approached quite differently.

Another idealisation of the mathematical treatment—and this is the one I am most interested in—is that finding a partner is assumed to be one sided. The treatment we're considering here assumes that it is an employers market. It assumes, in effect, that when you decide that you want to date someone, he or she will agree, and that when you decide to enter a relationship with someone, again they will agree. This mathematical equivalent of wishful thinking makes the problem more tractable but is, as we all know, very unrealistic.

A natural way to get around this last idealisation is to stop thinking about your candidate pool as a row of wallflowers at a debutants' ball, and instead think of your potential partners as active agents engaged in their own search for the perfect partner. The problem, thus construed, becomes much more dynamic and much more interesting. It becomes one of coordinating strategies. There is no use setting your sights on a partner who will not reciprocate. In order for everyone to find someone to reciprocate their interest, a certain amount of coordination between parties is required. This brings us to game theory.

## The Game of Love

Game theory is the study of decisions where one person's decision depends on the decisions of other people.[2] Think of games like chess or tennis, where your move is determined, at least in part, by what you think the other player's response will be. It is important to note that games do not have to be fun and are not, in general, mere diversions. The cold-war arms race can be construed as a "game" (in this technical sense of game) between military powers, each second-guessing what the other would do in response to their "moves". Indeed, the cold war was the stage for one of the original and most important applications of game theory. The basic idea of game theory is quite simple and should be very familiar: a number of players are making decisions, each of which depend on the decisions of the other players.

It's probably best to illustrate game theory via an example. Let's start with *the stag hunt*. This game originates in a story, by the 18th century political philosopher Jean-Jacques Rousseau, of cooperative hunting.[3] In its simplest form, the game consists of two people setting out to hunt a stag. It will take the cooperation of both to succeed in the hunt, and the payoff for a successful stag hunt is a feast for all. But each hunter will be tempted by lesser prey: a hare, for example. If one of the hunters defects from the stag hunt and opportunistically hunts a passing hare, the defector will be rewarded, but the stag hunt will fail so the non-defector will not be rewarded. In decreasing order of preference, the rewards are: stag, hare, and nothing. So the cooperative outcome (both hunt stag) has the maximum payoff for each of the hunters, but it is unstable in light of the ever-present temptation for each hunter to defect and hunt hare instead. Indeed, hunting hare is the safer option. In the jargon of game theory, the cooperative solution of hunting stag is *Pareto optimal* (i.e. there is no outcome that is better for both hunters), while the mutual-defect solution is *risk dominant* (in that it does not leave you empty-handed if your fellow hunter decides to defect and hunt hare) but it is not Pareto optimal. That is, the cooperative solution is best for both hunters and given that the other party cooperates in the stag hunt, then you should too. But if the other party defects and hunts hare, then so should you. Most importantly, both these outcomes are stable since neither party will unilaterally change from cooperation to defection or from defection to cooperation (again, in the jargon of game theory, the mutual defect and cooperation solutions are *Nash equilibria*[4]). So, in particular, if you both play it safe and hunt hares, there seems no easy way to get to the mutually preferable cooperative solution of stag hunting. Cooperation seems both hard to achieve and somewhat fragile. This game is important because it is a good model of many forms of cooperative behaviour.[5]

Consider another example, just to get a feel for game theory: *the prisoner's dilemma*. The scenario here is one where two suspects are questioned separately by the police and each suspect is invited to confess to a crime the two have jointly committed. But there is not sufficient evidence for a conviction so each suspect is offered the following deal: if one confesses, that suspect will go free while the other serves the maximum sentence; if they both confess, they will both serve something less than the maximum sentence, if neither confesses, they will both be charged with minor offences and receive sentences less than any of those previously mentioned. In order of preference, then, each suspect would prefer (i) confess while the other does not confess, (ii) neither confess (iii) both confess, and (iv) not confess while the other confesses. Put like this, it is clear what you should do: you should confess to the crime. Why? Because, irrespective of what the other suspect does you will be better off if you confess. But here's the problem, if both suspects think this way, as surely they should, they will both end up with the second worst outcome (iii). As a pair, their best outcome is (ii)—this is Pareto optimal, since neither can do better than this without the other doing worse—but the stable solution is (iii) where both defect—this is the Nash equilibrium, since given that one confesses the other should too. Group rationality and individual rationality seem to come apart. Individual rationality recommends both confessing, even though this is worse for both parties than neither confessing. Again we see that defection (this time from any prearranged agreement between the suspects to not confess) is rewarded and cooperation is fragile.[6]

What has hunting stags and police interrogations got to do with dating—crude metaphors aside? First, these two games demonstrate how important it is to consider the decisions of others when making your own decisions. What you do is determined in part at least, by what the other players in the game do and vice versa. So too, with relationships. In fact, the stag hunt is a very good model of cooperation in a relationship. Think of cooperatively hunting stag as staying faithful in a monogamous relationship. All going well, this holds great benefits for both parties. But there is always the temptation for one partner to opportunistically defect from the relationship to have an affair. This is the "hunting hare" option. If both partners do this, we have mutual defection where both parties defect from the relationship in favour of affairs. This game-theoretic way of looking at things gives us a very useful framework for thinking about our original question of why people stay in monogamous relationships.

### Where Did Our Love Go?

We are now in a position to see one account of how monogamous relationships are able to persist. Sometimes it will simply be the lack of opportunity for outside affairs. After all, there's no problem seeing why people cooperate in hunting stags when there are no alternatives. The more interesting case is when there *are* other opportunities. According to the game-theory account we are interested in here, an on-going monogamous relationship is a kind of social contract and is akin to the agreement to mutually hunt stag. But what binds one to abiding by this contract when there are short-term unilateral gains for defecting? Indeed, it seems that game theory suggests defection as a reasonable course of action in such situations. If the chances of catching a stag (or seeing the benefits of a lasting monogamous relationship) are slim, defecting by opportunistically catching a hare (or having an affair) seems hard to avoid, perhaps even prudent. But we must remember that the games in question are not isolated one-off situations, and this is the key.

While defection in the prisoner's dilemma or the stag hunt may be a reasonable course of action if the situation in question is not repeated, in cases where the game is played on a regular basis, there are much better long-term strategies. For instance, both players will see the folly of defecting in the first game, if they know that they will be repeatedly playing the same player. A better strategy is to cooperate at first and retaliate with a defection if the other player defects. Such so-called tit-for-tat strategies do very well in achieving cooperation. If both players are known to be playing this strategy, they are more inclined to cooperate indefinitely. There are other good strategies that encourage cooperation in these repeated games but the tit-for-tat strategy illustrates the point. In short, cooperation is easier to secure when the games in question are repeated, and the reason is quite simple: the long-term rewards are maximised by cooperating, even though there is the temptation of a short-term reward for defection. It's the prospect of future games that ensures cooperation now. Robert Axelrod calls this "the shadow of the future"[7] hanging over the decision. This shadow changes the relevant rewards in a way that ensures cooperation.

We can make the cooperative outcome even more likely and more stable by sending out signals about our intentions to retaliate if we ever encounter a defector. In the stag hunt, we might make it very clear that defection by the other party will result in never cooperating with them again in a stag hunt. (Translated into the monogamous relationship version, this amounts to divorce or sleeping on the couch for the rest of your life.) We might even make such agreements binding by making the social contract in question public and inviting public scorn on defectors. All this amounts to a change in the pay-offs for the game so that defection carries with it some serious costs; costs not present in the simple one-off presentation with which we started.

It is interesting to notice that this is pretty much what goes on in the relationship case. We have public weddings to announce to our friends and the world the new social contract in place (thus increasing the cost of a possible defection); we, as a society, frown on extra-marital affairs (unless they are by mutual consent); and most important of all, we are aware of the long-term payoffs of a good, secure, long-term, monogamous relationship (if, indeed that is what is wanted).

Now it seems we have the makings of an explanation of such relationships in terms of self interest. While cooperation might look as though it has to do with love, respect, ethics, loyalty, integrity and the like, the game-theory story is that it's all just narrow self interest. It's not narrow in the sense of being shortsighted, but in the sense that there's no need to consider the interests of others, except in so far as they impact on oneself. As

David Hume puts it: "I learn to do service to another, without bearing him any real kindness; because I foresee that he will return my service, in expectation of another of the same kind".[8] In particular, there seems to be no place for love (and acting out of love) in the account outlined here.

## Love is Strange

If all I've said so far is right, it looks as though we can explain faithful relationships in terms of narrow self interest. It's a case of "this is good for me, who cares about you". According to the game theory story, a faithful relationship is just a particular form of social cooperation. And all that is needed to keep the cooperation in place is mutual self interest. It has nothing to do with right or wrong, or caring for your partner. It's all in the game and the focus on payoffs to the individual—or at least, payoffs to the individual plus the shadow of the future. We might still frown upon non-cooperation, but not for the reasons usually assumed. We, as a society, frown on defectors because that's also part of the game and it's an important part of what is required to keep cooperation alive in the society at large.

You might be sceptical of all this. You might think that people fall in love and enter a relationship, not because they can get something out of it but … well, why? If you're not getting something out of it, surely you're doing it wrong! OK, perhaps you get something out of it but you stay committed through the hard times, through the arguments, through your partner's bad moods, not *purely* because it's good for you; you stick with them because they need you and you're a good person, right? It might help if that's what you believe, but one take-home message from the account I'm offering here is that there's no need for anything outside the game. We don't need to entertain anything other than self interest as a motivation for monogamous relationships. It may well be that it's useful to believe in such things as loyalty, goodness and perhaps even altruism, but they might all be just useful fictions—a kind of make believe that's important, perhaps even indispensable, but make believe all the same.

Let's look at these issues in terms of ethics. The game theory account not only leaves no room for love and romance, it also seems to leave ethics out of the picture. You might think that staying faithful is *ethically right* and engaging in extra-marital affairs is *unethical*. In so far as game theory says nothing about ethics, it would seem that it cannot be the whole story. But we can take this same game-theoretic approach to ethics. Ethics can be thought of as a series of cooperation problems. Thus construed, ethics is arguably explicable in the same terms.[9] The idea is that ethical behaviour is just stable, mutually-beneficial behaviour that is the solution to typical coordination problems (basically ethics is just a matter of "don't hurt me and I won't hurt you"), and societies that have robust solutions to such coordination problems do better than those that don't have such solutions. As in the relationship case, it might be beneficial to engage in the pretence that some actions really are right and some really are wrong, but again such pretence will be just a further part of the game. This new twist about ethics either makes your concerns about the dating and relationships case a lot worse or a lot better, depending on your point of view. On the one hand, this broader game-theoretic story about ethics allows that there is room for ethics in dating and relationships. But the ethics in question is just more game theory.

The picture of relationships I'm sketching here might seem rather different from the one we find in old love songs and elsewhere. I think the difference, though, is more one of emphasis. Think of the picture offered here as a new take on those old love songs rather than a different kind of song altogether. All the usual ingredients are here but in an unfamiliar form. We have fidelity, but it's there as a vehicle for serving self interest; ethical considerations are also there but they too are not what they first seem. I suggested that all the romance and ethics might be merely a kind of make believe, but perhaps that's overstating the case. The pretence may run very deep and it plausibly has a biological basis. If this is right, the game theory picture can be seen as offering insight into the true nature of romantic relationships. Love, for instance, is a commitment to cooperate on a personal level with someone, and licenses socially-acceptable forms of retribution if defection occurs. Perhaps this conception of love doesn't sound terribly romantic and is unlikely to find its way into love songs, but to my ears, this is precisely what all the songs are about—you just need to listen to them the right way. Love is less about the meeting of souls, and more about the coordination of mating strategies. If this makes love sound strange, then so be it: love is strange.

**Notes**

[1] See Thomas S. Ferguson, 'Who Solved the Secretary Problem?' (*Statistical Science*, Vol. 4, 1989, pp. 282–296) for more on the secretary problem and Clio Cresswell, *Mathematics and Sex* (Sydney: Allen and Unwin, 2003) for the many fascinating connections between mathematics and relationships.

[2] For classic treatments of game theory see: John von Neumann and Oskar Morgenstern, *Theory of Games and Economic Behavior* (second edition, Princeton: Princeton University Press, 1947) and R. Duncan Luce and Howard Raiffa, *Games and Decisions: Introduction and Critical Survey* (New York: John Wiley and Sons, 1957).

[3] Jean-Jacques Rousseau, *A Discourse on Inequality* (translated by M. Cranston, Penguin Books, New York, 1984, original from 1755).

[4] Named after John Nash, the subject of the Sylvia Nasar book *A Beautiful Mind* (New York: Simon and Schuster 1998) and the Ron Howard movie of the same name (and based on the Nasar book).

[5] Brian Skyrms, *The Stag Hunt and the Evolution of the Social Contract* (Cambridge: Cambridge University Press, 2004). This is an excellent treatment of the stag hunt and its significance for social cooperation.

[6] William Poundstone, *Prisoner's Dilemma* (New York: Doubleday, 1992). This is a very accessible introduction to the prisoner's dilemma and game theory. It outlines the origins of game theory in the RAND corporation during the cold war (with a frightening application to the nuclear arms race).

[7] Robert Axelrod, *The Evolution of Cooperation* (New York: Basic Books, 1984).

[8] David Hume, *A Treatise of Human Nature* (ed. L.A. Selby-Bigge, Oxford: Clarendon, 1949, original from 1739) p. 521.

[9] Richard Joyce, *The Evolution of Morality* (Cambridge, MA: MIT Press, 2006).