Mind Association

Heaps of Gluts and Hyde-ing the Sorites

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1. Introduction

The Sorites Paradox, like any paradox, affords a few basic options:

(a)   Accept the argument as sound.

(b)   Reject the argument as unsound due to:

   (i) invalidity;

   (ii) faulty premiss.

Philosophers attracted to (b) are usually attracted to so-called *gappy* semantics, semantics according to which some sentences are neither true nor false. The most prominent gappy approach is supervaluational semantics. On this approach the Sorites argument is deemed valid but a premiss is rejected as false.[1]

   Another approach to option (b) is so-called *glutty* semantics, semantics according to which some sentences are both true and false. Where gappy theorists see vagueness as generating *gaps*, glutty theorists see vagueness as generating *gluts*. Interestingly, glutty approaches to Sorites are about as *un*popular as gappy theories are popular.

   But why should this be? In his recent paper, Dominic Hyde (1997) focuses precisely on this question. His paper reveals a telling symmetry between gappy and glutty theories, and, in particular, between the supervaluational and so-called subvaluational semantics which respectively underwrite each theory. Hyde shows that a simple reinterpretation of standard supervaluational semantics gives rise to Jaskowski's (1969) pioneering paraconsistent semantics, a glutty semantics which is tailor-made for modelling vagueness. That Jaskowski's system is so suitably tailored is plain; as Hyde's paper shows, it is simply a dualised variant of the popular supervaluational model.[2] Accordingly, Hyde's

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[1] For space considerations we assume familiarity with the literature, and thus leave details aside.

[2] The formal details of this dualisation are sketched by Hyde but detailed by Achille Varzi (1994; 1995).

paper shows that the imbalance of attention given to gappy over glutty theories is unwarranted; any reason for the former is mirrored as a reason for the latter, and any reason for rejecting the latter is mirrored as a reason for rejecting the former.

Hyde is careful to point out that he is *not* claiming

> that the paraconsistent approach is *the* approach one ought to take to vagueness, but rather the weaker claim that many have pointed to its possibility and that with a dualisation of the very popular supervaluational semantics one can see exactly how such an approach can plausibly proceed. [Subvaluational validity] provides a semantics and logic of vagueness as good as [supervaluational validity] and, in the absence of conclusive grounds for rejecting truth-value gluts out of hand, there is no justification for the obscurity of the former approach in light of the latter's status as a preferred theory. (pp. 657–658, emphasis in original)

We think that Hyde has made significant steps towards his goals; he has shown that the paraconsistent approach has been unduly neglected, and he has indicated at least one route along which the paraconsistent approach might travel. We think, however, that there are at least two points which have been overlooked. In keeping with the theme of symmetry there is a point to be made in favour of gluts and a point in favour of (supervaluational) gaps. We make each point in turn.

2. From lying gluts to heaps of gluts

Hyde concerns himself with showing only that paraconsistent approaches to vagueness are in most noted respects as good as paracomplete approaches—approaches according to which some sentences may be gappy without all sentences being gappy. We think he could have gone further; glutty theorists have an ally which may well tip the scales in their favour.

The ally of glutty theorists is the family of so-called *paradoxes of self-reference*, including the liar, Russell's, Richard's, and so on.[3] As in the Sorites case, the paradoxes of self-reference have attracted both gappy and glutty responses. Moreover, as in the Sorites case, the overwhelming majority of philosophers have tended towards the gappy side. In *this* debate, however, such tendencies against gluts are difficult to maintain. As Sainsbury (1987) and Priest (1998a) have argued, there is no a

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[3] Given Yabloesque (Yablo 1993) paradoxes, the jury remains out as to whether the so-called paradoxes of self-reference are *merely* so called. For discussion see Sorensen (1998). For present purposes this issue can be set aside.

priori reason for rejecting gluts. But, then, given the following familiar considerations, glutty theories would seem to have the upper hand.

Paradoxes of any stripe are philosophically significant only in as much as they tell us about natural language. The paradoxes of self-reference are no different. Such paradoxes, as Tarski (1936; 1944) pointed out, appear to tell us that (say) English is inconsistent, that is, for some English sentence, A, both A and $\neg A$ are true, where '$\neg$' represents English negation. The big question is whether such appearances are accurate. Glutty theorists say 'yes' whilst (non-glutty) gappy theorists say 'no'.

This is not the place for a review of non-glutty 'solutions' to the paradoxes of self-reference. The main point is that all extant non-glutty 'solutions' to such paradoxes fall prey to strengthened paradox or, for independent reasons, fail to be theories of *natural* language.[4] Given that glutty approaches do *not* succumb to strengthened paradox but do save the natural-language appearances (in addition to offering desirable completeness), glutty theories appear to be preferable to gappy theories—at least in the debate over the paradoxes of self-reference.

Our point, now, is straightforward. Hyde shows that in the Sorites case, paraconsistent approaches are apparently just as good as their most popular rivals. What we wish to note is that there is an extra argument in favour of paraconsistency; the argument stems from the paradoxes of self-reference. In short, the paradoxes of self-reference push us towards gluts and, thereby, paraconsistency. Gappy approaches, but not glutty approaches, founder on strengthened (circularity) paradoxes. Since a 'solution' is not a solution to such paradoxes unless it solves the strengthened cases, glutty solutions are preferable. This, we think, tips the scale in favour of a paraconsistent approach to the Sorites Paradox. If nothing else, considerations of beauty and simplicity motivate a unified theory, with a unified underlying logic, of natural language. These virtues push towards a glutty theory of the entire language if they push for any part; there is reason to have a glutty theory for part of our language (viz. paradoxes of self-reference) and, ergo, for all. The point is simply that if there is good reason to think that we are already committed to a glutty logic and this logic can deal with the Sorites just as well as a gappy logic can,

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[4] We trust that this debate is familiar to regular readers of *Mind*, and so we won't go into the details. For relevant reading, see McGee (1997), Priest (1987), Sainsbury (1995), and Simmons (1993).

there is no reason to push for a gappy logic as well. Glutty logics can do it all.[5]

Now we turn to a point against Hyde's proposal.

3. Hyde-ing the Sorites

As the quotation in §1 makes clear, Hyde's aim is two-fold; he wants to show that paraconsistent approaches to Sorites have been wrongly neglected, and he wants to indicate 'exactly how such [a paraconsistent] approach can plausibly proceed' (pp. 657–658). As above, we think Hyde has accomplished the first (main) aim; however, there is reason to think that perhaps the second aim has not been met.

Hyde's proposed approach is beautifully straightforward. Let *SbV* be the particular paraconsistent semantics proposed. Hyde points out that, in *SbV*, *modus ponens* is not valid for *material* conditional (aka *the single arrow* → or *hook*, ⊃). This is to be expected. After all, $A \to B$ is equivalent to $\neg A \vee B$. But, now, suppose that A is glutty—that is, both A and $\neg A$ are true. Suppose, further, that B is not true (false only). Then A and $A \to B$ are true but B fails to be true, in which case *modus ponens* fails.[6] With the failure of *modus ponens* comes Hyde's proposed solution: Given that the Sorites is run using the material conditional and *modus ponens*, the paradox is accordingly blocked. Quite simply, the Sorites argument is not valid.

So goes Hyde's proposed approach. We do not dispute that *modus ponens* fails for the arrow when gluts are allowed, and so we agree that the Sorites, confined to arrow-like conditionals, is solved—where arrow-like conditionals are the material conditional and gussied up material conditionals like Lewis's fish-hook (strict implication). What we wish to emphasize, however, is that Hyde's proposal leaves open a very big and controversial issue. Hyde wants to show how a subvaluational treatment of the Sorites Paradox (in natural language) might go. What he fails to note is that whether his approach will work depends on whether there are ponenable conditionals (conditionals satisfying *modus ponens*) that may be used in Sorites reasoning.

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[5] Of course, if there are independent reasons elsewhere in the philosophy of language to commit us to gappy logic, we may once again face a stand-off between the gappy and glutty approaches to the Sorites. Exploring that issue, however, will take us too far afield. We are content here to demonstrate a point in favour of the glutty approach and to suggest that considerations elsewhere in the philosophy of language may well decide the matter of how best to deal with the Sorites.

[6] Of course, for precisely the same reason Disjunctive Syllogism fails in *SbV*; that is, we can have A and $\neg A \vee B$ both true but B false.

Let '⇒' be an entailment-expressing (logical implication) conditional, and let '→' be the material (disjunction-expressing) 'conditional'.[7] Most philosophers seem to think that English contains both sorts of conditional, though perhaps the latter is a conditional only by courtesy. Furthermore, let a *Sorites-supporting conditional* be a conditional that gives rise to Sorites Paradox; that is, such a conditional seems to be true when its antecedent and consequent are, for example, '*n* grains makes a heap' and '*n*-1 grains makes a heap'. The issue at hand, the crucial issue neglected in Hyde's paper, may now be seen as follows.

Virtually all philosophers, including Hyde (we presume), impose a minimal constraint on semantics for '⇒', namely that it satisfies *modus ponens*, or equivalently that it is ponenable or detachable. Accordingly, Hyde's approach to Sorites Paradox will not work if '⇒' is a Sorites-supporting conditional; in that case, one need merely run the Sorites argument with '⇒' instead of '→'. This, though, is not a problem. After all, '⇒' is not a Sorites-supporting conditional; it is not at all plausible that *that n grains makes a heap* entails *that n-1 grains makes a heap*. On the other hand, '→' is Sorites-supporting; but given the existence of gluts, disjunctive syllogism (material *modus ponens*) fails, and so '→' is not ponenable. Provided, then, that English contains no more than merely '⇒' and '→', Hyde's proposal may well be viable. The pressing question, however, is whether English contains a further, middle-strength conditional. That is, is there a conditional in English that is stronger than '→', weaker than '⇒', but is none the less both ponenable and Sorites-supporting?

The issue is very big and very controversial, and in this paper we will not settle it or attempt to settle it.[8] What we wish to emphasize is that Hyde's proposal stands or falls on the issue. After all, if there is such a ponenable, Sorites-supporting conditional in English, then Hyde's 'solution' is no solution at all; it merely hides the problem. The situation is decidedly different for the gappy (supervaluational) approach. Unlike Hyde's approach, the gappy approach is not affected at all by the existence of such a middle-strength conditional. If there is such a conditional, the gappy solution applies to it in just the way it applies to '→'.

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[7] Like most philosophers today, we follow the advice of Anderson and Belnap (1975, appendix) on the issue of treating 'entails' as a connective.

[8] For discussion see Lycan (forthcoming) and Priest (1987). See also Read (1995, chapter 3) and Jackson (1991).

4. Do we even need a conditional?

Finally we should say a little about why we have focused on the conditional version of the Sorites Paradox. After all, the paradox can apparently be generated without a conditional. For example, you might think that the paradox can simply be generated by a problematic series of questions (asking at successive stages: 'Is this a heap?').[9]

There are a couple of things to say in response to this. First, by far the most common presentation of the paradox employs a conditional and we are simply following suit.[10] Moreover, Hyde explicitly presents the paradox in this way and in so far as we are discussing Hyde's proposal, we feel obliged to focus on the conditional version. Still, the point remains: if the paradox can be presented without a conditional, isn't both Hyde's and our discussion beside the point?

We believe that despite claims to the contrary, versions of the Sorites that apparently make no reference to a conditional nevertheless require one. After all, the heart of the paradox is that after admitting an n-grained collection as a heap, it is very difficult to resist also admitting an $(n-1)$-grained collection as a heap. The best way, we believe, to spell this out is in terms of a conditional.

Take, for example, the suggestion that the paradox can be generated by a series of questions: Is this n-grained collection a heap?; Is this $(n-1)$-grained collection a heap?; and so on. Granted, one may find oneself puzzled in an effort to answer all the questions; however, as it stands, there is no genuine paradox here. There is only a paradox if we reply 'yes' to each question. But why do that? Why not say, for instance, that an n-grained collection is a heap, an $(n-1)$-grained collection is not a heap, an $(n-2)$-grained collection is a heap, an $(n-3)$-grained collection is not a heap? The problem with the question-version of the paradox is that it says nothing about how our answer to one question constrains our answer to the next question. We agree, of course, that the meaning of the word 'heap' rules out (we take it) the above alternating series of answers. Moreover, the fact that the word 'heap' is vague ensures that *if* we answer 'yes' to any of the questions, *then* we should not answer 'no' to the very next question. *Now* we can generate a paradox (so long as we answer 'yes' to one of the questions); but notice that we've had to employ a conditional to do it.

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[9] We thank an anonymous reader for *Mind* for this suggestion. This section is presented in response to the given reader's worry.

[10] See, for example, Read (1995), Sainsbury (1995), and Williamson (1994). It is not universally presented in terms of a conditional though. See, Priest (1991; 1998b) for identity versions of the Sorites argument. Discussion of the latter, however, would take us too far afield.

5. Closing remarks

Hyde's paper shows convincingly that paraconsistent approaches to Sorites Paradox have been wrongly neglected. We have argued that Hyde should have gone further; considerations of other paradoxes, coupled with the desire for a unified theory of natural language, independently motivate a paraconsistent treatment of the Sorites. We have also argued, however, that considerations of natural language may well tell against Hyde's particular paraconsistent proposal. Specifically, if, as seems plausible, English has a ponenable Sorites-supporting conditional, then the spectre of Sorites Paradox remains. To what extent the spectre should arouse concern remains to be seen.

Hyde has undoubtedly taken the debate forward. For good or bad, however, there are heaps of issues left unresolved.[11]

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