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Fuzzy Sets and Threatened Species Classification

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Introduction

Fuzzy set theory is a tool with great potential for dealing with uncertainty in conservation biology. In a recent publication in this journal, Todd and Burgman (1998) presented an interesting proposal for incorporating uncertainty and reliability into point-scoring methods of classifying threatened species. In particular, they proposed a fuzzy set-theoretic approach to the method of Millsap et al. (1990). There are, however, problems with this proposal. In the interest of developing such applications of fuzzy set theory in conservation biology, we think it is important to recognize and understand the shortcomings of the proposal advocated by Todd and Burgman (1998) so that such problems might be rectified in future developments.

We focus largely on two issues: (1) Todd and Burgman obtain fuzzy membership functions from probabilities, and the argument provided for this strategy is demonstrably fallacious, and (2) given that Todd and Burgman obtain their membership functions from probabilities and advocate fuzzy set-theoretic operations that are analogues of probability operations, the excursion into fuzzy set theory is not necessary.

Probabilities and Fuzzy Membership Functions

Fuzzy set theory was introduced to deal with vague predicates such as "tall" and "mature" (Zadeh 1965), that is, predicates that permit borderline cases (Sorensen 1989). The crucial idea in fuzzy set theory is that, unlike classical set theories, either axiomatic (Enderton 1977) or naive (Halmos 1960), an element can have a degree of membership between 0 and 1 in a given fuzzy set. This is expressed via a membership function (or characteristic function), that is, a map from the universe of discourse to the interval [0,1]. In classical set theories an element is either in or out of any given set—its membership value is either 1 or 0, respectively. Obtaining membership functions for fuzzy sets is subjective and nontrivial. The approach advocated by Todd and Burgman is the use of probabilities. In short, the question What is the probability that x belongs to some set X? is treated as equivalent to the question What is the degree of membership of x in X?

Todd and Burgman do not appreciate the difference between these two questions. For example, when they first introduce fuzzy sets they claim that "[f]uzzy sets allow for the inclusion of an element in a set when the membership of that element is unclear" (Todd & Burgman 1998:969). This is incorrect as a definition of fuzzy sets (Zadeh 1965; Williamson 1994; Read 1995; Zimmerman 1996). It may be that fuzzy sets can be employed to model uncertainty in these ways, but the membership is not unclear; it is just that an element can have a degree of membership. For instance, it is unclear whether $2^{120503} - 1$ is prime or not, but the set of prime numbers is still a classical set. This number is either in the set of prime numbers or it is not, even though it is unclear which.

Apart from the obvious philosophical differences between the two questions, there are other problems with treating probability of membership as a degree of membership. As Zadeh (1965) and Kandel and Byatt (1978) point out, there are various similarities between probability distributions and fuzzy membership functions—they both yield real numbers between 0 and 1, they both can be subjective, and, perhaps, both can be used to model uncertainty in data—but they are not the same thing (Zadeh 1965).

One difference is that probabilities are additive and fuzzy memberships are not. To see this, let the probability that an element x is in the set A be a (i.e., $P(x \in A) = a$). The probability that x is in the complement of A, then, is $P(x \in \overline{A}) = 1 - a$. The probability that x is in A or $\overline{A}$ is then $P(x \in A \cup \overline{A}) = P(x \in A) + P(x \in \overline{A}) = a + (1 - a) = 1$. With fuzzy membership values, the situation is entirely different. If the degree of membership of x in some fuzzy set B is b (i.e., $\mu_B(x) = b$)) and the degree of membership of x in the complement of B is $\mu_{\overline{B}}(x) = 1 - b$, then the degree of membership of x in B or its com-

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plement is not, in general, 1 (i.e., $\mu_{B \cup \bar{B}}(x) \neq 1$). Moreover, the degree of membership of x in B and $\bar{B}$ is not, in general, 0 (i.e., $\mu_{B \cap \bar{B}}(x) \neq 0$). These features of fuzzy set theory are not like probability theory. Given all this, why believe probabilities can be used as membership functions?

The argument provided by Todd and Burgman (1998) as justification for using probabilities as membership values runs, in essence, like this: probabilities are fuzzy measures and fuzzy measures are fuzzy membership functions; therefore, probabilities are fuzzy membership functions. This argument fails because a fuzzy measure is a map from the power set of the domain of discourse (i.e., the set of all subsets of the domain of discourse) to the interval [0,1]. That is, if g is a fuzzy measure, Ω is the universe of discourse and $\wp(\Omega)$ is the power set of Ω,

$$g : \wp(\Omega) \rightarrow [0,1].$$

A fuzzy membership function, on the other hand, is a map from the domain of discourse to the interval [0,1]. That is, if μ_A is a membership function for the set A and once again Ω is the universe of discourse, we have

$$\mu_A : \Omega \rightarrow [0,1].$$

The objects in the domain of g are sets of objects from Ω, whereas the objects in the domain of μ_A are objects from Ω. The two constitute entirely different domains, so μ_A is not the same as g. In short, a fuzzy measure on a set is not a fuzzy membership function for that set.

Given that the argument discussed above is the only justification provided by Todd and Burgman for using probabilities as fuzzy membership functions, a serious question now hangs over their whole strategy. Moreover, we see no way of fixing the argument in question. We are not, however, claiming that probabilities cannot be used to generate fuzzy membership functions, just that it is a mistake to identify fuzzy membership functions with probabilities in the way Todd and Burgman suggest.

There is reason to be skeptical, however, about the whole strategy of using a fuzzy set-theoretic approach to the Millsap categories. These categories are, after all, not fuzzy. Todd and Burgman try to avoid this difficulty by setting up the method in terms of belief. For example, the Millsap category "having population less than. . ." is replaced with the category "believed to have a population less than. . . ." Ultimately, however, the uncertainty associated with random variables (i.e., probabilistic uncertainty) is confused with the uncertainty associated with vagueness. It seems that the measurements of, or the probabilistic uncertainties about, the variables are being treated as fuzzy but the sets in question—the Millsap categories—are sharp. This suggests that a better approach to this whole problem might be to introduce fuzzy categories in place of the Millsap categories (as Regan et al. [1999] do for the IUCN categories) or to employ rough sets (Pawlak 1991). We won't pursue either of these suggestions here. We simply mention them as alternative proposals worthy of investigation.

Fuzzy Sets and their Operations

The second problem with Todd and Burgman's proposal is that the machinery of fuzzy set theory is unnecessary. The same results can be obtained with standard probabilistic methods, and these methods are ultimately preferable because (1) they avoid the unnecessary dressing of fuzzy set theory and (2) they are more familiar to working biologists.

Recall that the relevant probabilities are taken by Todd and Burgman (1998) as fuzzy membership values. To use fuzzy sets on the Millsap point-scoring method, it is necessary to perform the operations of intersection and union on these sets. It turns out there are many ways to define these operations in fuzzy set theory. Furthermore, these different operations do not (in general) produce the same results (as demontrated by Table 2 of Todd and Burgman 1998. Thus, it is important to choose the fuzzy set-theoretic operations carefully and to justify whatever operations are used.

After entertaining the usual minimum and maximum operations, Todd and Burgman suggest that the algebraic product and sum operations are more appropriate in this application. The latter operations, however, are analogues of the corresponding probabilistic operations. But because Todd and Burgman's fuzzy membership values are just probabilities in disguise, this approach is equivalent to a probabilistic treatment: exactly the same results could have been obtained without the excursion into fuzzy set theory. Moreover, there are good reasons for avoiding this excursion, chief among them being the fact that probabilistic methods are well understood by biologists and are more transparent. For example, once it is realized that the Todd and Burgman proposal is equivalent to a standard probabilistic approach, it is clear that it ignores dependencies between the various Millsap categories. This fact is not obvious in the fuzzy set-theoretic guise of the proposal.

There is also the issue of choosing the appropriate fuzzy set-theoretic union and intersection operations. Without some justification of why particular operations are used in a given application, the application remains questionable. Why, for instance, do Todd and Burgman claim that the algebraic product and sum have more appeal than the usual minimum and maximum operations? This is one aspect of a deeper issue concerning the interpretation of the mathematics invoked: abstract mathematical machinery is used to model data about biological systems without providing an interpretation of the machinery in question. Without an interpretation it is not clear what the mathematical results mean for the biological system under consideration.

Conclusion

We have drawn attention to two shortcomings of the fuzzy set-theoretic proposal for incorporating uncertainty and reliability into the assessment of threat put forward by Todd and Burgman (1998). We stress that these are shortcomings of the proposal as it stands: they are not necessarily shortcomings of all such fuzzy set-theoretic approaches to quantifying uncertainty and reliability in the assessment of threat. (For a fuzzy set-theoretic approach to addressing uncertainty in the IUCN categories that avoids these shortcomings, see Akçakaya et al. 2000). It is in the interests of further developments along these lines that we draw attention to the problems of the proposal of Todd and Burgman (1998). It is clear, however, that care needs to be exercised in using fuzzy set theory for such purposes.

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Literature Cited

Akçakaya, H. R., S. Ferson, M. A. Burgman, D. Keith, G. Mace, and C. R. Todd. 2000. Making consistent IUCN classifications under uncertainty. Conservation Biology 14(4):1001-1013.

Enderton, H. B. 1977. Elements of set theory. Academic Press, New York.

Halmos, P. R. 1960. Naive set theory. Van Nostrand, Princeton, New Jersey.

Kandel, A., and W. J. Byatt. 1978. Fuzzy sets, fuzzy algebra and fuzzy statistics. Proceedings of the Institute of Electrical and Electronics Engineers **66**:1619-1639.

Millsap, B. A., J. A. Gore, D. E. Runde, and S. I. Cerulean. 1990. Setting priorities for the conservation of fish and wildlife species in Florida. Wildlife Monographs **111**:1-57.

Pawlak, Z. 1991. Rough sets: theoretical aspects of reasoning about data. Kluwer, Dordrecht, The Netherlands.

Read, S. 1995. Thinking about logic: an introduction to the philosophy of logic. Oxford University Press, Oxford, United Kingdom.

Regan, H. M., M. Colyvan, and M. A. Burgman. 1999. A proposal for fuzzy IUCN categories and criteria. Biological Conservation **92**: 101-108.

Sorensen, R. A. 1989. The ambiguity of vagueness and precision. Pacific Philosophical Quarterly **70**:174-183.

Todd, C. R., and M. A. Burgman. 1998. The assessment of threat and conservation priorities under realistic levels of uncertainty and reliability. Conservation Biology **12**:966-974.

Williamson, T. 1994. Vagueness. Routledge, London.

Zadeh, L. 1965. Fuzzy sets. Information and Control **8**:338-353.

Zimmerman, H.-J. 1996. Fuzzy set theory—and its applications. 3rd edition. Kluwer, Dordrecht, The Netherlands.