ELSEVIER

Biological Conservation 92 (2000) 101–108

BIOLOGICAL
CONSERVATION

www.elsevier.com/locate/biocon

A proposal for fuzzy International Union for the Conservation of Nature (IUCN) categories and criteria

Helen M. Regan [a,*], Mark Colyvan [b], Mark A. Burgman [a]

[a] *School of Botany, University of Melbourne, Parkville, VIC 3052, Australia*
[b] *School of Philosophy, University of Tasmania, Hobart, TAS 7005, Australia*

Received 12 November 1998; received in revised form 11 March 1999; accepted 19 March 1999

Abstract

The classification of endangered species uses categories "extinct in the wild", "endangered" and so on that are intrinsically vague. This vagueness presents various problems for those trying to classify species. The usual way of dealing with this vagueness is to eliminate it by providing precise definitions of the categories in question. In this paper we propose a fuzzy set-theoretic alternative that respects the inherent vagueness of the crucial categories without compromising the utility of the classification scheme. Moreover, we argue that it leads to intuitively more appropriate classifications in many cases. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Conservation status; Endangered; Extinct; Fuzzy set-theory; IUCN criteria; Threat; Vagueness

1. Introduction

There are two quite distinct kinds of uncertainty with which science must deal. The first, called *epistemic uncertainty* (Williamson, 1994), is the uncertainty arising from incomplete data, limitations of measurement accuracy, extrapolations, interpolations, and so on. It can be reduced (but never completely eliminated) by improving one's data: by taking more comprehensive surveys, improving the accuracy of measuring devices, and so on. The second, called *vagueness* (following Sorensen, 1989; Williamson, 1994), is the uncertainty that arises from the fact that many of our natural language words [as opposed to *formal* languages, such as the language of first order predicate calculus where such problems do not arise (Jeffrey, 1991)], including a great deal of our scientific vocabulary, are vague in the sense that they permit borderline cases. For instance, reference to the number of "mature adults" in a population is vague since there are some individuals, adolescents, that are neither mature adults nor are they not mature adults. Adolescents are borderline cases with respect to the category "mature adults". The uncertainty that arises from vagueness is particularly resilient and, unlike epistemic uncertainty, is not reduced by improving one's data.

It is important to recognise and distinguish these two types of uncertainty since each needs to be handled differently. We must, therefore, learn to deal with each in the most appropriate manner. In this paper we are concerned primarily with the second type of uncertainty — vagueness — and, in particular, its bearing on the classification of endangered species via the International Union for the Conservation of Nature (IUCN) categories (IUCN, 1996). We argue that the current IUCN criteria can be improved by respecting the vagueness inherent in the crucial categories — vulnerable, endangered, critically endangered, extinct in the wild and extinct. We outline how current IUCN categories and criteria deal with vagueness in an unsatisfactory way. We suggest one approach — a fuzzy set-theoretic approach — and show how this may be used to incorporate the current criteria to yield better results in borderline cases. An example is provided of how one might construct a fuzzy set for vagueness in the description of the IUCN category "extinct". This proposal retains the spirit and the methodology of the present IUCN categories, but at the same time provides a better way of dealing with the vagueness inherent in the categories in question.

---

* Corresponding author. Tel.: +61-3-9344-6856; fax: +61-3-9347-5460. Current address: Applied Biomathematics, 100 North County Road, Setauket, NY 11733, USA.

E-mail address: h.regan@botany.unimelb.edu.au (H.M. Regan)

PII: S0006-3207(99)00060-9

2. Vagueness in conservation biology

Epistemic uncertainty can be reduced by improvements in the quality and quantity of the available data. It is clear, however, that (at least for practical purposes) some epistemic uncertainty will always be present. Because of these two facts it is extremely important to accurately specify the epistemic uncertainty in any data set. Moreover, there are various well-established and reliable methods available for this task. These include confidence intervals (Mendenhall et al., 1990), degrees of belief (Horwich, 1982) and imprecise probabilities (Walley, 1991). With vagueness the situation is different on two counts: (i) no amount of improvement of the data is going to tell us whether a particular borderline case ought to count as falling under the concept in question or not (Sorensen, 1989) and (ii) although there are various methods designed to deal with vagueness, there is no clear consensus on which is the best. Vagueness is known to lead to paradox — the so-called Sorites Paradox (Williamson, 1994; Sainsbury, 1995). This makes the need for an adequate method of dealing with it all the more pressing.

Consider the vague concept ‘endangered’. To most people, and most biologists, it means to be in danger of extinction, to be vulnerable to loss. One way of creating a tractable definition of a vague concept such as ‘endangered’ is to simply draw a line. So that a species is deemed endangered if it has less than n members, then removing one member will turn a non-endangered species into an endangered species if its original population is n (Sainsbury, 1995). The term ‘endangered’ is now a technical term defined to mean “less than n members”, quite a different meaning to that found in a dictionary or understood by most people. There are various problems associated with this approach, not least of which is that one cannot use the (technical) term “endangered” unless one is in possession of fairly accurate population data. Another more serious worry is that the technical term is *verbally* identical but not identical *in meaning* to the non-technical usage of the word. This also leads to problems. For instance, there is the practical problem of how to reconcile the technical usage with the everyday usage when the need arises. In the every-day usage of the word ‘endangered’, a difference in population size of one individual would not change the classification of a species (unless that difference leads to extinction), however, it can change the classification with the technical usage of the word.

To illustrate, let us consider an example, simplified for clarity. Suppose some agency is willing to spend money on saving endangered (in the common usage of ‘endangered’) species. The reason this money is allocated is that it is felt that endangered species have some significant probability of becoming extinct and that it is desirable to reduce this risk. Resources for conservation are always limited and there are competing demands on their allocation. In these circumstances, it is important to identify those species that are endangered and those that are not, this being one of the parameters that determines the allocation of funds for protection of species. What is to be done with borderline cases? If the technical sense of ‘endangered’ is adopted, there are no borderline cases so there is no problem. But the money allocated is for those species that are endangered in the common usage of the word, not in the non-vague technical sense. The non-vague technical sense says nothing about being at risk of extinction, for instance. So either the two must be reconciled, by making non-scientific language precise as well, or the vagueness in common usage must be dealt with. We suggest the latter, for amongst other reasons, it is important that the general public not be alienated from the decision making procedure.

3. Approaches to dealing with vagueness

Although the regimentation of scientific language to rule out vague terms has had some notable supporters, including Carnap (1950), Frege (1960), Haack (1978; 1996) and Quine (1981), serious questions hang over such a program. It is contentious whether an ideal scientific language, free of vagueness, is possible. Moreover, in such a language there would be no room for vague concepts like ‘patch’, ‘clump’, ‘plant’, or ‘ecosystem’. It might reasonably be concluded from this that such an ideal theory denies the existence of such categories as plants, ecosystems and so on. This is a serious violation of common sense (Hyde, 1998). In any case, most scientists would be disinclined to follow such a route because the classifications of interest — vulnerable, endangered, critically endangered, extinct in the wild and extinct — are important for (amongst other things) political and social purposes. They are used to set priorities for conservation attention and funding, to elicit donations and votes in the political arena, and the number of endangered species is used as a benchmark in environmental reporting. Interest in whether a given species is endangered is not solely due to scientific considerations. Listing protocols include point-scoring systems (Millsap et al., 1990; Lunney et al., 1996), rule sets (IUCN, 1994; Keith, 1998) and qualitative procedures (Master, 1991).

Bertrand Russell (1923) suggested that a great deal of both scientific and everyday language is vague and that such vagueness cannot be eliminated. His solution was to deny that vague language fell within the scope of classical (bivalent) logic. Many logics have been put forward as contenders for the replacement logic. These include extensions of classical logic, such as various modal logics (Williamson, 1994, pp. 270–275; Hughes and Cresswell, 1996), paraconsistent logic (Hyde, 1997), and van Fraassen's method of supervaluations (van Fraassen, 1966; Fine, 1975; Dummett, 1992) but we will focus on *multi-valued logics*: these give up the classical principle of bivalence.

Early writers appealed to various versions of three-valued logic (Hallden, 1949; Körner, 1955) and more recently Putnam (1983) has suggested intuitionistic logic for the task of dealing with vagueness. However, these all seem to suffer technical difficulties, perhaps the most serious of which is that such logics still propose sharp divisions where intuitively none occur (Williamson, 1994, pp. 102–113). In effect, a three-valued logic carves the territory up into three categories. So, for example, the predicate 'endangered' is partitioned into the categories 'truly endangered', 'borderline endangered' and 'falsely endangered'. Intuitively we feel that there cannot be a sharp cut-off between these three categories and yet that is exactly what the three-valued logic yields. Thus, it seems three-valued logics simply postpone the problem, they do not solve it. This same difficulty faces any finite-valued logic. This leads, rather naturally, to the proposal of an infinite-valued logic such as the continuum-valued system of Lukasiewicz and Tarski (1930). This proposal is given its most popular form by Zadeh (1965; 1975) with his so-called *fuzzy logic* or, alternatively, *fuzzy set theory*.

The central idea of fuzzy set theory is that elements of sets can have *degrees of membership*. This is in stark contrast with classical set theory [both naïve (Halmos, 1974) and the axiomatic theories such as Zermelo-Fraenkel (Enderton, 1977)]. More formally, the membership function of classical set theory is a map from the universe to the two element set $\{0,1\}$, whereas the fuzzy set-theoretic membership function is a map from the universe to the closed interval $[0,1]$. Thus, fuzzy set theory (or the Lukasiewicz–Tarski continuum-valued logic) can be used as the base logic for fuzzy logic, in which the range of the membership function is identified with (countably) infinitely many fuzzy truth-values (Haack, 1978, pp. 162–169). For the remainder of this paper we will explore a fuzzy set theoretic approach and its consequences for the IUCN categories.

4. A fuzzy set-theoretic approach to the IUCN categories

There are two potential problems with the IUCN categories as they stand. The first is illustrated in the example we presented earlier of an agency allocating funds to endangered species. It may turn out that the populations of two species differ by one, and yet one species is classified as endangered and the other not. Consequently, one receives funding while the other receives none.

We are not claiming that such sub-optimal classifications are encouraged by the IUCN or that such classifications are common (in fact, they seem to be very rare). We are merely pointing out that such classifications are possible given the present IUCN rules, and this is enough to suggest that the present rules might be improved upon. (See IUCN 1996, p. intro 21 for a useful summary of the existing categories and criteria.)

The problem is that there really ought not to be sharp boundaries between the various IUCN categories. Indeed this is recognised by the IUCN:

> ...there is no clear line that separates threatened and non-threatened species. There is in fact a continuum, and we have to choose appropriate points at which to divide one group from another. (IUCN, 1996, p. intro 17)

The mistake, we claim, is in thinking that "we must choose appropriate points at which to divide one group from another" (IUCN, 1996, p. intro 17). Fuzzy boundaries allow the separation of threatened and non-threatened species *without* providing sharp cut-off points.

The second problem is that the creation of precise thresholds for the various endangered categories, as the IUCN have (IUCN 1994; 1996), is of little use unless we have sufficiently accurate data to allow the various distinctions to be drawn. For example, if a species has a population declining at a rate of at least 50% in 10 years (or three generations) it is, according to the IUCN criteria, *endangered*. If the population is declining at a rate of at least 80% in 10 years (or three generations), it is *critically endangered*. To decide if a given species is critically endangered, data are required to differentiate rate of population decline, for instance, 79 and 80%. Such accuracy is rarely possible. This means that there is likely to be a great deal of misclassification of, or inability to classify, species in these border areas. In practice, uncertainty close to the boundaries of these classifications is resolved by applying the precautionary principle, under which the person classifying a species will err on the 'safe' side and classify the species as critically endangered unless reasonably sure it is not. But such decisions are not always transparent, they are subject to individual interpretations of reasonable safety, and they raise the spectre of the manager being unable to distinguish between species that are 'definitely' endangered and those that are only 'perhaps' endangered. Fuzzy boundaries, however, are more forgiving with imprecise data. A 1% difference in population-decline rates will not mean the difference between being critically endangered or not; it will just mean that the species in question is classified as very slightly more or very slightly less critically endangered.

It now remains to provide some fuzzy criteria for the fuzzy set-theoretic approach. That is, some (fuzzy) membership functions must be provided for the various categories. This task is both subjective and non-trivial.

We propose the following general strategy. Firstly, we see this proposal as a refinement of the existing IUCN criteria. Thus, we incorporate the existing criteria as midpoints in the fuzzy membership functions. That is, the membership functions must yield values of 0.5 for the existing boundaries. Secondly, some expert opinions on the limits of the borderline cases are required. For example, with the critically endangered category discussed earlier, an estimate is required of how far above and below the present demarcation (and our mid-point of the membership function) of 80% we must go before the species is definitely critically endangered and definitely not critically endangered respectively. Let us suppose that $(80+a)\%$ is the least value such that a species is definitely critically endangered and that $(80-b)\%$ is the greatest value such that the species is definitely not critically endangered. (Here a and b are positive real numbers). A membership function, μ, for the set "critically endangered" may thus be constructed as follows:

$$\mu = (x) = \begin{cases} 0 & x < 80 - b \\ \dfrac{x}{2b} + \dfrac{b-80}{2b} & 80 - b \leqslant x < 80 \\ \dfrac{x}{2a} + \dfrac{a-80}{2a} & 80 \leqslant x \leqslant 80 + a \\ 1 & x > 80 + a \end{cases}$$

This is represented in Fig. 1 in graph form.

In general the membership functions will be of the form:

$$\mu(x) = \begin{cases} 0 & x < d - b \\ \dfrac{x}{2b} + \dfrac{b-d}{2b} & d - b \leqslant x < d \\ \dfrac{x}{2a} + \dfrac{a-d}{2a} & d \leqslant x \leqslant d + a \\ 1 & x > d + a \end{cases} \tag{1}$$

where d is the current demarcation and a and b are the upper and lower bounds, respectively, of the borderline region.

Of course many other membership functions are possible. For instance, it may be desirable to simplify the function suggested above by setting a equal to b. Or it may be preferable to choose a smooth function [the one presented above has cusps at $x=(80-b)$, $x=80$ (unless $a=b$) and $x=(80+a)$]. But even this first attempt is an improvement on a sharp boundary. The reason it is better is that a species estimated to have declined by 82% in 10 years is critically endangered to degree 0.51, whereas, a species estimated to have declined by 78% in 10 years is also critically endangered, but to a lesser degree — to degree 0.49 in this case. When making decisions about the allocation of resources, this additional information may be carried with the classification quite naturally, providing the manager with an additional highly relevant piece of information with which to support the decision making process. The resulting classification for each species would have two pieces of information: the class to which a species belongs, and the degree to which it belongs in that class.

The present proposal, while useful as a ranking method, provides more information than ordering schemes. Ranking methods may, for example, rank

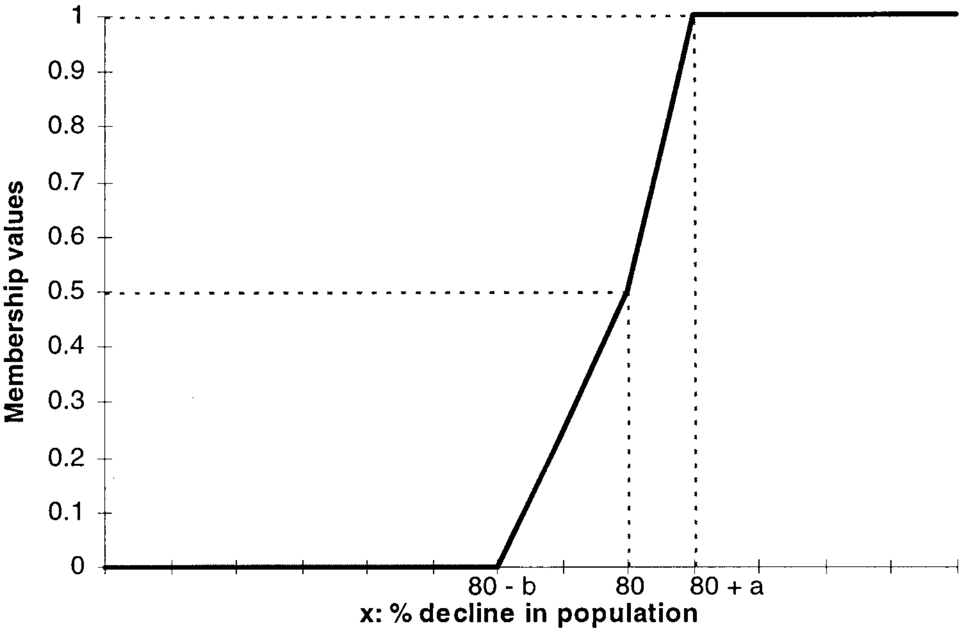

Fig. 1. Membership function for the critically endangered category.

species within a category based on an ordinal scale. The proposal advocated in this paper provides information about the distance between the various items in addition to the ranking. Just as knowing the exact locations of Brisbane, Sydney and Hobart provides more information than simply knowing how they are arranged from North to South, knowing the degree to which three species are endangered provides more information than a simple ordering from most to least endangered. For example, a species deemed to be threatened to a degree of 1 (on a scale from 0 to 1), a second species to a degree of 0.9 and a third to a degree of 0.1 is more informative than an ordinal ranking of 1, 2 and 3 from most threatened to least threatened. Moreover, this extra information may allow for more reliable decisions about conservation priorities and funding.

It is worth noting here that the expert opinion required for the construction of this membership function is concerned with the use of the term 'critically endangered'. That is, the vagueness inherent in the term is of interest and experts are required to provide some guidance on its usage. In this respect they are not requested to provide information on their estimates of the actual populations or degree of risk faced by individual species. In short, they are required to provide estimates of the extent of the vagueness of the crucial categories, not estimates of epistemic uncertainty.

5. Fuzzy set-theoretic approach for the IUCN category "extinct".

There are some apparent differences between the case discussed above and some of the other categories, in particular, those that deal with the various notions of extinction. Firstly, if the category in question is sharp, then the membership function will be a classical one. For example, it's plausible that the category "extinct" is sharp: a species is extinct if and only if the species has no members. The resulting membership function will yield 1 for any such species and 0 for all others. There is, however, still the question of how to determine whether a species has no members. This is an *epistemic* matter-it is not a matter of vagueness in the category "extinct".

Determining whether or not a species is extinct, however, is problematic. The IUCN state that a species is extinct when there is *no reasonable doubt* that the last individual has died. There is no reasonable doubt that the hadrosaurs (duck-billed dinosaurs) are extinct because they were large and would be easy to see, they have not been observed for millions of years, their habitat has changed to such an extent that they no longer exist and the fossil record supports the hypothesis that a catastrophic event occurred that wiped them out.

The *Caloprymus campestris* (desert rat-kangaroo) is also listed as extinct because there is no reasonable doubt that the last individual has died. There is, however, more uncertainty in the claim that *C. campestris* is extinct than with the same claim for the hadrosaurs. There has been less time since the last observation of the rat-kangaroo, there has been a less dramatic reduction in their habitat and the desert rat-kangaroos are/were much smaller, and therefore more difficult to observe. These are all elements of epistemic uncertainty. The term "no reasonable doubt" allows species to be classified as extinct when there is a small degree of uncertainty in the classification. There are two issues at play here. The first is that there is a threshold of epistemic uncertainty that is allowed to constitute "no reasonable doubt". For instance, it may be decided that a species is extinct if it is suspected that there is a 99% chance that it is extinct even though there is a 1% chance that this could be wrong. The second issue is that the term "no reasonable doubt" is vague - there are degrees of "no reasonable doubt". So while the set of extinct species is a genuinely sharp one, the terminology used in deciding if a species is extinct is vague.

There have been many instances where a species has been recorded as extinct and then rediscovered (Table 1). These highlight the problem of knowing whether a species is truly extinct. The best that can be done in attempting to determine whether or not a species is extinct is to offer guidelines that allow the inference that a species is likely to be extinct. One piece of relevant information is the time since the last observation (Smith et al., 1993). One current convention is to conclude that a species has become extinct if there has not been a recorded observation for 50 years. This time span is arbitrary and suffers from the problem that it is applied to all species regardless of the frequency with which the species was observed in the period before the last observation.

Species that were observed frequently and then not at all for many years are more likely to be extinct than species that were observed only seldomly and then not at all for the same period of time and with the same search effort. Solow (1993) and then Burgman et al. (1995) addressed this by calculating the probability of extinction as a function of the time since the last observation, the frequency of observations in the past and the period of time from the first observation to the last observation.

Consider the schematic representation of frequency data for a species in Fig. 2. Each cell represents a regular time period and each circle represents an independent observation within the given time period. Blank cells indicate that there was no observation in that time period. Solow (1993) provides a formula for the probability that a species is still extant when only single observations are made in a cell. Burgman et al. (1995) modified Solow's equation to allow for multiple observations in a cell. The probability that a species is extant is calculated as

Table 1
Some examples of species of Australian animals that have been rediscovered in the past 35 years[a] (after Short and Smith, 1994; Burgman and Lindenmayer, 1998)

Species	Last record before rediscovery	Year of rediscovery
Mountain Pygmy Possum	± 15 00 years BP[b]	1966
Parma Wallaby	1932	1966
Leadbeater's Possum	1909	1961
Dibbler	± 1884	1967
New Holland Mouse	± 1887	1967
Sandhill Dunnart	1894	1969
Bridled Nailtail Wallaby	± 1930s	1972
Long-tailed Dunnart	± 1940s	1984
Gilbert's Potoroo	1869	1994
Greater Stick-nest Rat	1938	1986
Night Parrot	1912	1979
Noisy Scrub-bird	1889	1961

[a] The table does not include species for which there was some doubt of the species taxonomic status and later revision resulted in a taxon being partitioned into two or more species [e.g. the highly endangered Mahogany Glider (Van Dyck, 1993)].

[b] BP = years before present.

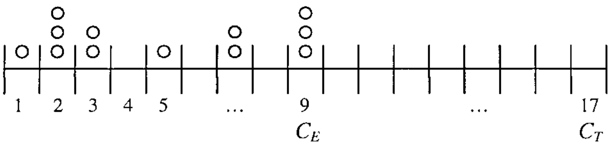

Fig. 2. A schematic representation of observations (represented as circles) of a species in regular time intervals. Here the total number of time intervals is $C_T = 17$, the number of time intervals between the start of the observation period and the last observation is $C_E = 9$ and the total number of observations is $N = 12$.

$$p = \left(\frac{C_E}{C_T}\right)^N \tag{2}$$

where C_T is the total number of cells (these could be years or months or any uniform time interval), C_E is the number of time intervals between the start of the observation period and the last observation and N is the total number of observations. For the same number of sightings, N, and the same observation period, C_T, a longer absence of observations at the end of the entire observation period will give a higher probability of extinction than for a shorter run of absences. For species with the same values for C_E and C_T, the species with the higher number of observations, N, will have a higher probability of extinction.

To calculate bounds on the time since last observation that constitutes no reasonable doubt that a species has become extinct, we investigated collection data of 190 *Acacia* species from Western Australia. Burgman et al. (1999) analysed these collection data to assess the efficacy of a number of methods in detecting changes and trends in the conservation status of taxa (cf. Grimson et al. 1992; McCarthy 1998). Collection data of Western Australian conservation *Acacia* species were extracted from the specimen data base of the Western Australian Herbarium (see Burgman et al., 1999 for details).

The expected time since the last observation required to give a 99% chance of extinction was calculated for each of the 190 species using Eq. 2. This gave values ranging from 5 years to 286 years since the last observation. We have assumed that a likelihood of extinction any lower than 99% constitutes a reasonable doubt that the species is extinct. Hence we conclude the range of times since last observation for which we have no reasonable doubt that a species is extinct is from 5 to 286 years since the last observation. Eq. 1, with $b = 5$, $d = 50$ and $a = 286$ provides a membership function for the fuzzy set associated with the time since last observation constituting no reasonable doubt that a species has become extinct. It is a simple matter to amend the graph in Fig. 1 to accommodate this function.

When endeavouring to categorise a species as extinct or not it is prudent to use Solow's equation directly to determine the probability that the species is extinct, along with any other relevant information such as habitat decline, if the data exist to do so. In such cases it is not recommended to resort to the fuzzy set constructed here. In many cases, however, this type of data is not available. Sometimes the only information available is the time since the last observation. In the absence of exhaustive collection records, when there is no indication of how frequently a species might have been observed, had there been a collection effort, the fuzzy set constructed above provides a means of classifying a species to some degree of "no reasonable doubt" of extinction. Burgman et al. (1995) provide examples of two animal species, the black-footed ferret and the Caribbean seal, where 50 years since the last observation is too long a wait before the former should be classified as extinct, and too short a wait for the latter. While there are some species that have a 99% chance of extinction and others that may have a close to zero chance of extinction at the lower end of the range of values for time since last observation calculated from the *Acacia* collection data, the fuzzy set construction proposed here is an attempt at incorporating the range of times since last observation that we would have to wait for most species to be classified as extinct.

We must emphasise that the membership values between zero and one for the fuzzy set constructed here do not specify the degree to which a species is extinct because the set of extinct species is a sharp set. Instead, they specify the degree to which there is no reasonable doubt that a species, particularly one for which there is very little collection data, is extinct. There may be other, more sensible ways of calculating the bounds on the vague region comprising no reasonable doubt that the last individual of a species has died. We have presented one possible way of producing the membership function for the vagueness inherent in the problematic case of how to decide if a species is extinct.

A second apparently different type of category is what might be called the *non-numerically vague*. Take, for example, the category 'extinct in the wild'. There is undoubtedly vagueness about what counts as 'in the wild': animals counted as in the wild may include animals on large reserves, animals in open range zoos, domesticated animals within their natural range or wild animals outside their natural range (such as camels in arid Australia).

Although the category "extinct in the wild" is vague it does not permit a natural ordering of the graduations from the clear cases to the clear non-cases in the same way that, say, the endangered category does. In the latter category, the relevance of the number of individuals, decline rates and so on invoke natural orderings which are absent in the former case. The lack of obvious natural orderings in the non-numerically vague categories makes it difficult — but not impossible — to produce membership functions for them. For example, one way of tackling the problem is to use some sort of weighted point-scoring system to transform the non-numerically vague into the numerically vague. For example, a species could be allocated a score for each of the categories above, and weighted by the proportion of the total population in that state.

6. Operations on fuzzy sets

The primary advantage of a fuzzy category, such as our proposed critically endangered category, is that it avoids some otherwise intractable difficulties, while remaining faithful to the established IUCN categories. Firstly, since it is possible to speak of degrees of critically endangered, it provides us with the flexibility to allocate funds and resources that correspond to how critically endangered the species in question is. For example, an organisation responsible for the allocation of funds to threatened species could elect to allocate funds (in part) according to the degree to which taxa are critically endangered — spending more on those that are critically endangered to a high degree and less on the borderline cases. Secondly, if data are imprecise and it is only known that the species' population decline is between 78 and 82%, for instance, it will be classified as critically endangered between $3a/b + (b - 80)/2b$ and $41/a + (a - 80)/2a$ (supposing $b, a > 2$). It is still preferable to strive for precision, but in the mean-time the imprecision does not result in drastic misclassification of, or inability to classify, the species in question.

Since many of the IUCN criteria are conjunctions (logical AND statements) and disjunctions (logical OR statements) (IUCN, 1996 p. intro 21), it is necessary to explain how to combine fuzzy membership functions so they will be implementable within the framework developed by the IUCN. To do this a little of the technical machinery of fuzzy set-theory is needed. The rules required are for set-theoretic union (for the disjunctive criteria) and for set-theoretic intersection (for the conjunctive criteria). For two fuzzy sets, A and B, and the universal set, Ω, the following set-theoretic operations hold:

Union $\forall x \in \Omega, \mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$.

Intersection $\forall x \in \Omega, \mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$.
(Ross, 1997)

The first part of criterion B in IUCN (1996, p. intro 21) says that a species is critically endangered if either its extent of occurrence is less than 100 square kilometres or its area of occupancy is less than 10 square kilometres. Now suppose that a membership function for the extent of occurrence yields some value, ν, and a membership function for the area of occupancy yields μ. The disjunctive structure of the criterion means that the resulting set-theoretic structure is a union, so the maximum of ν and μ must be selected. This maximum is then the degree of membership of the extension of the disjunctive criteria presented in the first part of criterion B. The case for conjunctive criteria is similar. The set-theoretic analogue of conjunction is intersection, so the minimum is chosen of the two values in question. Care needs to be taken when using complicated criteria such as criterion C in IUCN, (1996, p. intro 21). This criterion consists of a conjunction in which the second conjunct is a disjunction.

This will involve taking both maxima and minima and the order is obviously important. Careful attention to the logical structure of the criteria will ensure the correct order of operations.

7. Conclusion

In this paper we have provided an outline of a fuzzy set-theoretic approach to the classification of endangered species. Our approach respects the original criteria to the extent that these are incorporated as mid-points in the new fuzzy membership functions. The only extra information required are some expert opinions on the extent of the borderline regions for each existing demarcation. From this information it is possible to construct fuzzy IUCN categories and criteria. We provided an example of how one might go about constructing a fuzzy set for the vagueness associated with classifying a species as extinct. In this example we used a method from the literature, namely Solow's modified equation (Solow 1993;

Burgman et al. 1995), to calculate bounds on the fuzzy membership function. This proposal has many advantages over the existing categories and criteria. These advantages arise from the fact that the demarcations between the categories in question are not sharp — and this fact is respected in our approach. Indeed the IUCN agree that there is no sharp boundary between the categories in question but for lack of a viable alternative, draw sharp boundaries anyway. Our proposal provides such an alternative without abandoning the work that has gone into arriving at the sharp boundaries found in IUCN (1996) and without abandoning the spirit of the IUCN classification scheme.

Acknowledgements

Some of the material from this paper was presented at the 12th Annual Meeting of the Society for Conservation Biology at Macquarie University in July of 1998. We thank the participants in the subsequent discussion for their contributions. We would also like to thank Dominic Hyde for useful discussions and for comments on an earlier draft of this paper. Finally we are indebted to an anonymous referee for many helpful suggestions.

References

Burgman, M.A., Grimson, R.C., Ferson, S., 1995. Inferring threat from scientific collections. Conservation Biology 9, 923–928.

Burgman, M.A., Maslin, B.R., Andrewartha, D., Keatley, M.R., Boeck, C., McCarthy, M., 1999. Inferring threat from scientific collections: power tests and an application to Western Australian *Acacia* species. In: Ferson, S., Burgman, M.A. (Eds.), Quantitative Methods for Conservation Biology. Springer-Verlag, New York.

Burgman, M.A., Lindenmayer, D.B., 1998. Conservation Biology for the Australian Environment, Surrey Beatty and Sons, Chipping Norton, NSW.

Carnap, R., 1950. The Logical Foundations of Probability, Chicago University Press, Chicago.

Dummett, M., 1992. Wang's paradox. In Dummett, M. Truth and Other Enigmas, 2nd impression, pp. 248–268. Duckworth, London (reprint).

van Dyck, S., 1993. The taxonomy and distribution of *Petaurus gracilis* (Marsupialia: Petauridae) with notes on its ecology and conservation status. Memoirs of the Queensland Museum 33, 77–122.

Enderton, H.B., 1977. Elements of Set Theory, Academic Press, San Diego.

Fine, K., 1975. Vagueness truth and logic. Synthese 30, 265–300.

Frege, G., 1960. In: Geach, P., Black, H. (Eds.), Translations from the Philosophical Writings of Gottlob Frege. Blackwell, Oxford, 2nd edition.

Grimson, R.C., Aldrich, T.E., Drane, J.W., 1992. Clustering in sparse data and an analysis of Rhabdomyosarcoma incidence. Statistics in Medicine 11, 761–768.

Haack, S., 1978. Philosophy of Logics, Cambridge University Press, Cambridge.

Haack, S., 1996. Deviant Logic Fuzzy Logic: Beyond the Formalism, University of Chicago Press, Chicago.

Halldén, S., 1949. The Logic of Nonsense, Uppsala Universitets Arsskrift, Uppsala.

Halmos, P.R., 1974. Naive Set Theory, Springer-Verlag, New York.

Horwich, P., 1982. Probability and Evidence, Cambridge University Press, Cambridge.

Hughes, G.E., Cresswell, M.J., 1996. A New Introduction to Modal Logic, Routledge, London.

Hyde, D., 1997. From heaps and gaps to heaps of gluts. Mind 106, 641–660.

Hyde, D., 1998. Vagueness ontology and supervenience. The Monist 81, 297–312.

IUCN, 1994. International Union for the Conservation of Nature. Red List Categories. IUCN Species Survival Commission, Gland, Switzerland.

IUCN, 1996. IUCN Red List of Threatened Animals. International Union for the Conservation of Nature, Gland, Switzerland.

Jeffrey, R., 1991. Formal Logic: Its Scope and Limits. 3rd Edition, McGraw-Hill, New York.

Keith, D.A., 1998. An evaluation and modification of IUCN Red List criteria for classification of extinction risk in vascular plants. Conservation Biology 12, 1076–1090.

Körner, S., 1955. Conceptual Thinking, Cambridge University Press, Cambridge.

Lukasiewicz, J. & Tarski, A., 1930. Untersuchungen über den Aussagenkalkül. Reprinted in Tarski, A. (J.H. Woodger Trans; edited by Corcoran, J.), 1983. Logic Semantics Metamathematics. 2nd Edition. Hackett, Indianapolis.

Lunney, D., Curtin, A., Ayers, D., Cogger, H.G., Dickman, C.R., 1996. An ecological approach to identifying the endangered fauna of New South Wales. Pacific Conservation Biology 2, 212–231.

Master, L.L., 1991. Assessing threats and setting priorities for conservation. Conservation Biology 5, 559–563.

McCarthy, M.A., 1998. Identifying declining and threatened species with museum data. Biological Conservation 83 (1), 9–17.

Mendenhall, W., Wackerly, D.D., Scheaffer, R.L., 1990. Mathematical Statistics with Applications, 4th Edition, PWS-Kent, Boston.

Millsap, B.A., Gore, J.A., Runde, D.E., Cerulean, S.I., 1990. Setting the priorities for the conservation of fish and wildlife species in Florida. Wildlife Monographs Supplement, Journal of Wildlife Management 54, 5–57.

Putnam, H., 1983. Vagueness and alternative logic. Reprinted in Putnam, H., Realism and Reason: Philosophical Papers Volume 3. Cambridge University Press, Cambridge, pp. 271–286.

Quine, W.V., 1981. What price bivalence? Reprinted in Quine, W.V., Theories and Things, Harvard University Press, Cambridge, Mass, pp. 31–37.

Ross, T.J., 1997. Fuzzy Logic with Engineering Applications, McGraw-Hill, New York.

Russell, B., 1923. Vagueness. Australasian Journal of Philosophy and Psychology 1, 84–92.

Sainsbury, R.M., 1995. Paradoxes. 2nd Edition, Cambridge University Press, Cambridge.

Short, J., Smith, A.P., 1994. Mammal decline and recovery in Australia. Journal of Mammology 75, 288–297.

Solow, A.R., 1993. Inferring extinction from sighting data. Ecology 74, 962–964.

Smith, F.D.M., May, R.M., Pellew, R., Johnson, T.H., Walter, K.R., 1993. How much do we know about the current extinction rate? Trends in Ecology and Evolution 8, 375–378.

Sorensen, R.A., 1989. The ambiguity of vagueness and precision. Pacific Philosophical Quarterly 70, 174–183.

van Fraassen, B., 1966. Singular terms truth value gaps and free logic. Journal of Philosophy 53, 481–495.

Walley, P., 1991. Statistical Reasoning with Imprecise Probabilities, Chapman and Hall, London.

Williamson, T., 1994. Vagueness, Routledge, London.

Zadeh, L.A., 1965. Fuzzy sets. Information and Control 8, 338–353.

Zadeh, L.A., 1975. Fuzzy logic and approximate reasoning. Synthese 30, 407–428.