CONCEPTUAL CONTINGENCY AND ABSTRACT EXISTENCE

By Mark Colyvan

Mathematical statements such as 'There are infinitely many prime numbers' and '$2^{\aleph_0} > \aleph_0$' are usually thought to be necessarily true. Not everyone is convinced of this, though. Hartry Field, for instance, thinks that all the statements of mathematics (except negative existential statements) are false because there are no mathematical objects. In this paper, I want to focus on the modal status of Field's claim – he claims that mathematical statements are *contingently* false. In particular, he claims that mathematical entities do not exist but that they might have. He is thus committed to *contingent nominalism*.[1]

Hale and Wright, on the other hand, believe in the necessary existence of mathematical entities.[2] They thus disagree with Field on all fronts, but in the papers I have cited they concentrate on the contingency/necessity issue. Before I discuss their objections to Field's position, it is worth pointing out that what is at stake here is the contingency of Field's contingent nominalism. Hale and Wright are just as unhappy with what we might call *contingent Platonism* (the view that mathematical entities exist but that they might not have). Indeed, Wright states this quite explicitly:

> Field has no prospect of an account of what the alleged contingency is contingent *on*. The world does not contain, in Field's view, but might have contained, numbers. But there is no explanation of *why* it contains no numbers; and if it had contained numbers, there would have been no explanation of that either. There are no conditions favourable for the emergence of numbers, and no conditions which prevent their emergence.[3]

The narrow focus of this debate on the contingency/necessity issue has prevented all participants from noticing what I take to be a rather important point in favour of Field: Hale and Wright's arguments do not work against certain versions of contingent Platonism.

Since much of the early debate has been conceded by the parties, I shall skip over those details and focus instead on the portion of the debate that is still live. A word or two, though, on the notion of contingency under discussion. Although Field (p. 285) maintains that his view only commits him to the *logical* possibility of the existence of mathematical entities, he has allowed the debate to continue in terms of *conceptual* possibility (see, e.g., p. 285). Here a statement is taken to be logically possible if it is not first-order logically contradictory; it is conceptually possible, roughly, if its negation is not true by virtue of its meaning. In the present discussion I too shall take the relevant notion of contingency to be conceptual rather than logical.

Now to the current debate. In a nutshell, Hale and Wright wonder what the contingent non-existence of mathematical entities might be contingent upon. Rather than press this point directly, however, they consider the curious notion of *insularity*. I shall call some proposition *brute* if whether or not it obtains does not depend on anything else, *barren* if no phenomena depend on whether it obtains or not, and *absolutely insular* if it is both brute and barren. If Field's programme of showing mathematics to be dispensable to science can be carried out,[4] then claims about

[1] H. Field, 'The Conceptual Contingency of Mathematical Objects', *Mind*, 102 (1993), pp. 285–99.

[2] R. Hale and C. Wright, 'Nominalism and the Contingency of Abstract Objects', *Journal of Philosophy*, 89 (1992), pp. 111–35, hereafter NCAO; and 'A *Reductio ad Surdum*? Field on the Contingency of Mathematical Objects', *Mind*, 103 (1994), pp. 169–84, hereafter RAS.

[3] Wright, 'Why Numbers Can Believably Be', *Révue Internationale de Philosophie*, 42 (1988), pp. 425–73, at p. 465.

[4] See Field, *Science without Numbers: a Defence of Nominalism* (Oxford: Blackwell, 1980), for details of his programme.

mathematical entities would appear to be absolutely insular. Absolute insularity, though, is metaphysically dubious.[5] This is something on which all the participants in the debate agree. The question is, what is the correct anti-insularity principle? Hale and Wright (NCAO pp. 133–4) suggest the following:

1. *Hale and Wright's Anti-insularity Principle.* There are no absolutely insular conceptual contingencies.

If Field's programme is successful, (1) ensures that the existence or non-existence of mathematical entities is not a matter of conceptual contingency, since in that case claims about mathematical entities would be absolutely insular.

Field, however, thinks that (1) is the wrong anti-insularity principle, and I am inclined to agree with him on this. He suggests (pp. 296–7) the following example:

> Call something a *surdon* iff
> (A) its existence and state are in no way dependent on the existence and state of anything else; and
> (B) the existence and state of nothing else are in any way dependent on the existence and state of it.
>
> This certainly seems to be a conceptually consistent concept; but (A) and (B) guarantee insularity, so [principle (1)] immediately guarantees the existence of surdons – indeed, the conceptual necessity of their existence. Of course, Hale and Wright accept this conclusion, since they take numbers to be surdons, but even they should baulk at the idea that establishing the existence of mathematical entities is as easy as this!

Field concludes that (1) cannot be right. The correct anti-insularity principle he suggests is

2. *Field's Anti-insularity Principle.* We should not (at least not without very compelling reason) assume the existence of absolutely insular entities.

Thus according to Field (p. 297), 'unless mathematical entities prove indispensable, we ought not to believe in them'.

To this Hale and Wright reply (RAS p. 180) that Field has misconstrued their original argument:

> Our objection was not that Field's view of the (putative) fact that there are no numbers infringes an evidently acceptable principle [(1)] but that it forces an unmotivated decision against an attractive principle.

But surely Field's belief in the contingent existence of mathematical entities is not 'unmotivated'. He has very good theoretical grounds for believing in this, and he makes them clear in a number of places. Furthermore, he does not rule (1) out of play without offering a suitable replacement, namely, (2). What is more, his example of surdons demonstrates a point in favour of his own (2) over Hale and Wright's (1).

Hale and Wright (RAS pp. 181–3) also do not agree that on their account Platonism is obtained as easily as Field suggests in the above passage. They claim that

[5] See my 'Can the Eleatic Principle be Justified?', *Canadian Journal of Philosophy*, 28 (1998), pp. 313–36, for the related issue of the alleged dubiousness of causally idle entities.

conceptual consistency only defeasibly justifies claims of possibility. Thus the conceptual consistency of surdons does not guarantee their necessary existence. Even granting Hale and Wright this, it seems that (2) is at least as plausible as (1), and so Field is not guilty of 'an unmotivated decision against an attractive principle'.

Where does this leave us, then? Hale and Wright claim that contingent nominalism would violate (1), and since the latter is an attractive metaphysical principle it ought not to be forced out of play without good reason. But now if we take a step back from the narrow focus of the debate so far, we see that Hale and Wright's (1) does not rule out a contingent Platonism supported by Quinean indispensability considerations.

According to the Quine–Putnam indispensability argument, we ought to believe in mathematical entities because of the role they play in our best physical theories.[6] The existence of such entities is clearly not absolutely insular. Indeed, some have even suggested that on the Quinean view mathematical entities are causally active.[7] Moreover, on this view, mathematical entities are on an ontological par with other theoretical entities, and so it is quite natural to see their existence as contingent. So it seems that Hale and Wright's (1) rules out contingent nominalism, but not contingent Platonism (at least not the Quinean version of contingent Platonism). Given that Hale and Wright are quite clear that it is the contingency that they find unpalatable (at least in the present context),[8] it seems that (1) fails to achieve what was required of it.

There is no parallel problem for Field in this respect. Field has no quarrel with either contingent Platonism or contingent nominalism (for present purposes), and indeed his (2) does not rule against either of these positions. So if one is suspicious of absolute insularities, as I think one should be, then it seems that his anti-insularity principle is to be preferred over Hale and Wright's. And if one accepts Field's principle, both contingent nominalism and contingent Platonism are live options.

Now it might be argued that it is not contingency *simpliciter* that Hale and Wright find objectionable, but *unexplained* contingency (in particular, unexplained contingent nominalism and contingent Platonism). If this is right, it is not so clear that Hale and Wright ought to be unhappy with the Quinean position I sketched above. For it might be argued that according to this view the existence of mathematical entities is not an unexplained contingency: it is contingent upon the obtaining of certain physical theories. Since the contingency in question is not unexplained, it should not be ruled out by Hale and Wright's anti-insularity principle. Thus (1) does indeed deliver the desired result. But there is good reason to suspect that Hale and Wright would not accept this argument. The reason is that an analogous argument seems to

[6] For details, see H. Putnam, 'Philosophy of Logic', repr. in his *Mathematics, Matter, and Method: Philosophical Papers*, Vol. 1, 2nd edn (Cambridge UP, 1979), pp. 323–57; W.V.O. Quine, 'On What There Is', repr. in his *From a Logical Point of View*, 2nd edn (Harvard UP, 1980), pp. 1–19.

[7] For a discussion of this issue, see C. Cheyne and C. Pigden, 'Pythagorean Powers or a Challenge to Platonism', *Australasian Journal of Philosophy*, 74 (1996), pp. 639–45; M. Colyvan, 'Is Platonism a Bad Bet?', *Australasian Journal of Philosophy*, 76 (1998), pp. 115–19.

[8] See Hale, *Abstract Objects* (Oxford: Blackwell, 1987), p. 110; also the passage from Wright's 'Why Numbers Can Believably Be' quoted on p. 88 above.

show that they ought not object to (Field's) contingent nominalism either. Let us suppose that (i) Field has successfully nominalized some physical theory Γ, but some alternative theory Δ doggedly resists nominalization; and (ii) Γ turns out to be the correct theory of our world. Suppositions (i) and (ii) together give us reason to believe nominalism to be true. What is more, this nominalism would be contingent upon the obtaining of the physical theory Γ.

It is not clear what Hale and Wright ought to say to this. They have a number of options. (a) They might reject the claim that the obtaining or non-obtaining of a physical theory counts as a legitimate explanation (in the relevant sense) for the existence or otherwise of mathematical entities. In that case both (Field's) contingent nominalism and (Quine's) contingent Platonism are left unexplained, and so Hale and Wright face the problem that (1) does not rule out (Quinean) contingent Platonism as they would like. (b) They might reject the claim that it was only *unexplained* contingency they were unhappy with. Again this leaves them with (1) failing to do the job for which it was designed. (c) Finally, they might accept the arguments of the previous paragraph and concede that contingent nominalism and contingent Platonism are unobjectionable.[9]

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