Mathematics and Aesthetic Considerations in Science

MARK COLYVAN

Joseph Melia, in a recent paper in this journal (Melia 2000), outlines a very interesting nominalist strategy for the philosophy of mathematics. In essence, Melia argues, contra Putnam (1971), that it is not inconsistent nor intellectually dishonest to quantify over mathematical objects and yet deny the existence of such objects. Although I have reservations about this strategy (which Melia calls 'weaseling'), I will not dwell on them in this present paper.[1] The issue on which I wish to focus is Melia's suggestion that platonist theories are not simpler (in the appropriate sense) than their nominalist counterparts, and thus platonist theories do not enjoy any relative advantages over nominalist theories. I will discuss Melia's simplicity claim and I will show that although there is an important insight underlying it, there are applications of mathematics in physical science that prove problematic for Melia. The applications I have in mind do not seem to be merely examples of mathematics yielding more attractive descriptions of the world, but, rather, these applications seem to give us important insights into our world. Such applications, by Melia's own lights, present good reason to embrace platonism.

There are three main arguments in Melia's paper. The first is that the only way open for the nominalist is to deny that we ought to be ontologically committed to *all* the indispensable entities in our best scientific theories—that is, he argues that Hartry Field's (1980) nominalist project is doomed to failure. Melia's second argument is supposed to establish the coherence of an alternative strategy—this is Melia's weaseling strategy. What is lacking so far is a reason to embark on such a nominalist strategy in the first place. But the motivation for this should be obvious to everyone: nominalist theories are (ontologically) more

[1] Although the details of Melia's strategy are novel, similar proposals have been put forward in recent years by Mark Balaguer (1996; 1998, ch. 7) and Jody Azzouni (1997a; 1997b). What all these approaches have in common is that each attempts to provide an easier way to nominalism than the difficult and rather technical path proposed by Hartry Field (1980).

Mind, Vol. 111 . 441 . January 2002

parsimonious than their platonist counterparts. The problem is that there is a rather unsatisfying standoff looming. Sure, says the platonist, nominalist theories are more parsimonious in that they posit fewer objects, but they lack the elegance of platonist theories. After all, the platonist continues, we must balance ontological parsimony with other theoretical virtues such as elegance, explanatory power, simplicity and the like.

Melia's response to this deadlock is insightful and worth careful consideration. He suggests that mathematics does not simplify theories *in the appropriate way*. He argues that mathematics allows for simpler theories of the world but this does not mean that, according to these theories, the world is a simpler place. Melia suggests that we should only prefer theories according to which the world is a simpler place—that is the kind of simplicity at issue. He admits, however, that:

> Of course, there *may* be applications of mathematics that do result in a genuinely more attractive picture of the world—but defenders of this version of the indispensability argument have yet to show this. And certainly, the defenders need to do more than point to the fact that adding mathematics can make a theory more attractive: they have to show that their theories are attractive in the right kind of way. That a theory can recursively generate a wide range of predicates, that a theory has a particularly elegant proof procedure, that a theory is capable of making a large number of fine distinctions are all ways in which mathematics can add to a theory's attractiveness. But none of these ways results in any kind of increase in simplicity, elegance or economy to our picture of the *world*. Until examples of applied mathematics are found that result in this kind of an increase of attractiveness, we realists about unobservable physical objects have been given no reason to believe in the existence of numbers, sets or functions. (Melia, 2000, pp. 474–5, emphasis in original)

There are two things to say in reply to Melia. The first is that to claim that theory T is simpler than theory T' but that the simplicity which T enjoys over T' is only in the theory and not in the world seems to beg important questions. But then again, to deny this may also be said to beg questions. I thus won't pursue this response, for I suspect that it too just ends in a standoff. In any case, there is a much more persuasive response to Melia: a response that directly meets the challenge he poses in the above passage.

I agree with Melia that the platonist must demonstrate that platonist theories are more attractive. But, Melia recognizes that being attractive is not just a matter of being simple (even though towards the end of the paper he focuses on simplicity). Indeed, when laying out a version of the indispensability argument (due to Quine) Melia explicitly mentions explanatory power, simplicity, and strength (Melia 2000, p. 456) Moreover, he apparently endorses these as theoretical virtues. I have argued elsewhere (Colyvan 1999; 2001a) that in some cases mathematics may be seen to add to the unificatory power and predictive power of theories and that this gives us some reason to prefer platonist theories over nominalist theories.[2] Along similar lines, John Burgess (1983) and Alan Baker (2001) argue that mathematics is required for scientific progress and again this tips the scales in favour of platonism. I won't rehearse all these arguments here. Instead, I'll outline one case that illustrates one way in which complex numbers may be said to unify a great deal of science.

Consider a physical system described by the differential equation:

(1) $y - y'' = 0$

(where y is a real-valued function of a single real variable). Equations such as these describe physical systems exhibiting (unconstrained) growth[3] and we can solve them with a little elementary real algebra.[4] But now consider a strikingly similar differential equation that describes certain periodic behaviour:

(2) $y + y'' = 0$

(where, again, y is a real-valued function of a single real variable). Somewhat surprisingly, the same real algebra cannot be used to solve (2)—we must employ complex methods.[5]

Now since complex algebra is a generalization of real algebra, we can employ the same (complex) method for solving both (1) and (2). Thus we see how complex methods may be said to unify, not only the mathe-

[2] Steiner (1989; 1998) also argues for the thesis that mathematics seems to be responsible for many novel predictions. Steiner, however, does not draw any ontological conclusions from this argument; indeed, Steiner believes that the applicability of mathematics presents problems for platonists and nominalists alike. I too have come to realise that the applicability of mathematics presents problems for both platonism and nominalism (Colyvan, 2001b). The fact remains, however, that the cases both Steiner and I discuss are exactly the kinds of cases that Melia challenges the platonist to produce.

[3] Growth is usually described by a first-order differential equation, but there is good reason to employ second-order equations such as this (Ginzburg, 1986).

[4] The details need not concern us here. I spell out some of the details in Colyvan (1999). See Boyce and DiPrima (1986) for a thorough treatment of elementary differential equations.

[5] Again I won't pause over the details.

matical theory of differential equations, but also the various physical theories that employ differential equations. But the unification doesn't stop there. The exponential function, which is a solution to (1), is very closely related to the sine and cosine functions, which are solutions to (2). This relationship is spelled out via the definitions of the complex sine and cosine functions. Without complex methods, we would be forced to consider phenomena described by (1) and (2) as completely disparate and, moreover, we would have no unified approach to solving the respective equations. I see this is a striking example of the unification brought to science by mathematics—by complex numbers, in this case. (It's by no means the only such case though; detours into complex analysis are commonplace in modern mathematics—even for what are essentially real-valued phenomenon.)

Examples such as this are problematic for any nominalists who subscribe to the view that unificatory power is a theoretical virtue. Such examples are even more worrying for those persuaded by Friedman (1974) and Kitcher (1981) to the view that scientific explanation is unification.[6] The latter, it seems, must accept that complex numbers are not just facilitating a means of simplifying the statement of various theories, but that complex numbers are genuinely explanatory—and there's no doubt that explanatory power is a theoretical virtue (at least for scientific realists). Even for those not persuaded by Friedman and Kitcher there may yet be cases where mathematics seems to be playing a crucial role in explanation. For example, the Minkowski geometric explanation of the Lorentz contraction in special relativity is, arguably, a non-causal explanation (indispensably) employing mathematical entities such as the Minkowski metric (Colyvan 1998).

Where does this leave us then? Platonist theories may have greater unificatory power (and perhaps greater explanatory power), while nominalist theories may be (ontologically) more parsimonious? Are we thus back at the unsatisfying stand off we faced at the beginning of this section? I suspect that depends on whether you have nominalist sympathies or platonist sympathies in this debate. In any case, my aim here has not been so ambitious as to settle the platonism–nominalism debate. I'm content to show that we platonists can meet Melia's worthy challenge. We do so by pointing to examples such as the one above and others in Steiner (1989; 1998), Colyvan (1999; 2001a), and Baker (2001). Such examples show that mathematised theories exhibit theoretical virtues well beyond the descriptive simplicity that Melia discusses (pp. 472–3), and that these virtues are not likely to be found in their nomi-

[6] For example, Field (1993) is attracted to this view of explanation.

nalist rivals.[7] Whether this is enough to justify the inflated ontology of platonism must, for the time being, remain an open question.[8]

School of Philosophy
University of Tasmania
GPO Box 252-41
Hobart, Tasmania, 7001
Australia
mark.colyvan@utas.edu.au

MARK COLYVAN

References

Azzouni, Jody 1997a: 'Applied Mathematics, Existential Commitment and the Quine-Putnam Indispensability Thesis'. *Philosophia Mathematica (3)*, 5, pp. 193–209.

——1997b: 'Thick Epistemic Access: Distinguishing the Mathematical From the Empirical'. *Journal of Philosophy*, 94, pp. 472–84.

Baker, Alan 2001: 'Mathematics Indispensability and Scientific Progress'. *Erkenntnis*, 55, pp. 85–116.

Balaguer, Mark 1996: 'A Fictionalist Account of the Indispensable Applications of Mathematics'. *Philosophical Studies*, 83, pp. 291–314.

——1998: *Platonism and Anti-Platonism in Mathematics*. New York: Oxford University Press.

Boyce, William E. and DiPrima, Richard C. 1986: *Elementary Differential Equations and Boundary Value Problems*. 4th edn. New York: John Wiley.

Burgess, John, P. 1983: 'Why I am not a Nominalist'. *Notre Dame Journal of Formal Logic*, 24, pp. 93–105.

Colyvan, Mark 1998: 'Can the Eleatic Principle be Justified?' *Canadian Journal of Philosophy*, 28, pp. 313–36.

——1999: 'Confirmation Theory and Indispensability'. *Philosophical Studies*, 96, pp. 1–19.

——2001a: *The Indispensability of Mathematics*. New York: Oxford University Press.

[7] I say 'not *likely* to be found in their nominalist rivals' because in most cases the nominalist theories are not yet available for comparison. I hope, however, to have said enough for it to seem plausible that whichever way the nominalist theory turns out, it will lack the unificatory power exhibited in the differential-equations case presented above.

[8] An earlier version of this paper was presented at the Society for Exact Philosophy meeting at the University of Montreal in May 2001. I'd like to thank the audience at that meeting for their many valuable comments. I'm also indebted to Otávio Bueno, Lev Ginzburg, and Mary Leng for reading earlier drafts of this paper. Their helpful suggestions led to many improvements.

——2001b: ‘The Miracle of Applied Mathematics’. *Synthese*, 127, pp. 265–78.

Field, Hartry 1980: *Science Without Numbers: A Defence of Nominalism*. Oxford: Blackwell.

——1993: ‘The Conceptual Contingency of Mathematical Objects’. *Mind*, 102, pp. 285–99.

Friedman, Michael. 1974: ‘Explanation and Scientific Understanding’. *Journal of Philosophy*, 71, pp. 5–19.

Ginzburg, Lev R. 1986: ‘The Theory of Population Dynamics: Back to First Principles’. *Journal of Theoretical Biology*, 122, pp. 385–99.

Kitcher, Philip 1981: ‘Explanatory Unification’. *Philosophy of Science*, 48, pp. 507–31.

Melia, Joseph 2000: ‘Weaseling Away the Indispensability Argument’. *Mind*, 109, pp. 455–79.

Putnam, Hilary 1971: *Philosophy of Logic*. New York: Harper.

Steiner, Mark 1989: ‘The Application of Mathematics to Natural Science’. *Journal of Philosophy*, 86, pp. 449–80.

——1998: *The Applicability of Mathematics as a Philosophical Problem*. Cambridge, Mass.: Harvard University Press.