Scientific Realism and Mathematical Nominalism: A Marriage Made in Hell

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The Quine-Putnam Indispensability argument is the argument for treating mathematical entities on a par with other theoretical entities of our best scientific theories. This argument is usually taken to be an argument for mathematical realism. In this chapter I will argue that the proper way to understand this argument is as putting pressure on the viability of the marriage of scientific realism and mathematical nominalism. Although such a marriage is a popular option amongst philosophers of science and mathematics, in light of the indispensability argument, the marriage is seen to be very unstable. Unless one is careful about how the Quine-Putnam argument is disarmed, one can be forced to either mathematical realism or, alternatively, scientific instrumentalism.

I will explore the various options: (i) finding a way to reconcile the two partners in the marriage by disarming the indispensability argument (Jody Azzouni [2], Hartry Field [13, 14], Alan Musgrave [18, 19], David Papineau [21]); (ii) embracing mathematical realism (W.V.O. Quine [23], Michael Resnik [25], J.J.C. Smart [27]); and (iii) embracing some form of scientific instrumentalism (Otávio Bueno [7, 8], Bas van Fraassen [30]). Elsewhere [11], I have argued for option (ii) and I won't repeat those arguments here. Instead, I will consider the difficulties for each of the three options just mentioned, with special attention to option (i). In relation to the latter, I will discuss an argument due to Alan Musgrave [19] for why option (i) is a plausible and promising approach.

From the discussion of Musgrave's argument, it will emerge that the issue of holist versus separatist theories of confirmation plays a curious role in the realism–antirealism debate in the philosophy of mathematics. I will argue that if you take confirmation to be an holistic matter—it's whole theories (or significant parts thereof) that are confirmed in any experiment—then there's an inclination to opt for (ii) in order to resolve the marital tension outlined above. If, on the other hand, you take it that it's a single hypothesis that's confirmed in a given experiment, then you'll be more inclined towards option (i). As we shall see, Musgrave's argument illuminates, in an

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interesting and original way, the important role confirmation has to play in realism debates in the philosophy of mathematics.

1 Scientific Realism Meets Mathematical Nominalism

Scientific realists such as Musgrave [20] are happy to go beyond what is observable and posit unobservable entities. According to scientific realists, what makes the cloud chamber appear as though there is an electron in it is that *there is and electron in it*. The details of how we go from mere observations, which typically underdetermine the theory, to the positing of unobservable entities varies. Inference to the best explanation is the vehicle of choice amongst most scientific realists. Indeed, it's not stretching things too much to suggest that the scientific realism–antirealism debate can be characterised in terms of the acceptance or rejection (respectively) of inference to the best explanation. In any case, Musgrave, like most scientific realists, accepts this much-discussed form of inference.[1]

Mathematical nominalism is the view that mathematical entities, such as, numbers, functions, and sets do not exist. The opposing view—mathematical realism—holds that at least some mathematical entities exist. One of the primary motivations for mathematical nominalism is that mathematical realism faces a rather daunting epistemological challenge [6]. The problem is simply that if mathematical entities exist, as the mathematical realist would have it, then we require an adequate account of how we come by knowledge of such entities. After all, mathematical entities, if they exist, do not seem to be the kinds of things that have space-time locations or have causal powers. In short, if they exist, it would seem we cannot have any contact with them and hence we cannot have knowledge of them. Nominalism does not face any vexing epistemological issues, so it seems more reasonable to suppose that mathematical entities do not exist. (Or so the argument goes.)[2]

[1] Of course scientific realism has its problems. For instance, justifying the clearly invalid inference to the best explanation and dealing with the underdetermination of theory by evidence. (See [29] for details.) I'll not dwell on such problems in what follows.

[2] Nominalism has some problems too. One we'll look at in the next section, but there is also the problem of supplying a uniform semantics across all natural and scientific language. The problem is simply that scientific sentences such as 'there's a planet closer to the sun than Venus' is true and what makes it true is the existence of Mercury (and the fact that it is closer to the sun than Venus). But nominalists hold that there are no numbers, so it would seem that the nominalist cannot employ the usual semantics to account for the truth of sentences such as 'there is a number smaller than 2'. (See [6] for further details.)

2 The Tension and the Options

At first glance, scientific realism and mathematical nominalism make a handsome couple. There's no need for belief in mysterious abstract mathematical entities, the epistemology is relatively straightforward, and there's a healthy respect for science, taken at face value. No reinterpretation of science in terms of observables or dodgy appeals to the world behaving as though there were unobservables. A little thought, however, soon reveals the problems with this union. The problem is that the alliance is very unstable. The scientific realism part of the marriage typically appeals to inference to the best explanation as a reason for belief in unobservable theoretical entities. But even a cursory glance at any scientific text, from almost any area of science will reveal that crucial role mathematics plays in science. We have mixed mathematical-empirical statements such as:

> (*) The work done in moving a body from a to b is given by $\int_a^b F(s)\, ds$, where F is the force exerted on the body and s is the body's displacement.

We also have purely mathematical statements such as:

> (**) The Gaussian distribution is symmetric about its mean.

Both kinds of statement play important, indeed, indispensable, roles in science. As Quine [23] and Putnam [22] have pointed out if one is to accept such statements as true (as surely we must), then this in turn leads us to accept the existence of real-valued force functions, integrals, displacement functions, Gaussian distributions, and means.

To summarise this line of thought, we ought to count as real any entity that plays an indispensable role in our best scientific theories. As Putnam has stressed, anyone inclined to do otherwise would be guilty of intellectual dishonesty. (This is the sin of "denying the existence of what one daily presupposes" [22, p. 347].) Following Putam, let's call this the indispensability argument.[3] This argument is usually construed as an argument for mathematical realism, but since it relies on certain background assumptions (such as naturalism and confirmational holism) it is not going to persuade everyone (at least not without a defence of its background assumptions). But notice that this argument counsels us to accept entities as real, irrespective of whether they are observable or unobservable. All that matters is that the entities in question are indispensable. But what does the latter involve?

One way an entity might play an indispensable role in a scientific theory is that it might be indispensable for explanation. That is, inference to the best explanation is a special case of the indispensability argument [14, pp. 14–20]. Moreover, as has already been noted, this is a style of argument

[3] I lay out this argument in more detail and defend it in [11].

that the scientific realist accepts.[4] In fact that's all we need; we don't really need to consider more general forms of the indispensability argument because mathematical entities surely feature prominently in various explanations.[5] (See (*) and (**) above, for instance, and consider the various scientific explanations such statements feature in.) So here I will take the indispensability argument to be an argument that puts pressure on the marriage of scientific realism and nominalism. It does this because the style of argument is one which scientific realists already endorse. Now let's consider the various options facing would-be nominalist scientific realists.

2.1 Marriage Counselling

By far the most popular option for dealing with the tensions I just outlined is to somehow reconcile scientific realism with mathematical nominalism. There are a number of different strategies proposed for this purpose. These divide into what I call "easy road" and "hard road" strategies. The easy road strategies involve denying that we ought to have ontological commitment to all the entities that are indispensable to our best scientific theories. That is, we provide some principled demarcation between those parts of our best scientific theories that are to be treated realistically and those which are to be treated instrumentally. And, of course, for this strategy to work, the mathematical entities had better fall on the instrumental side of the divide.[6]

Another way to proceed is to deny that mathematical entities are indispensable to our best scientific theories. This is a hard road since it involves showing how to do science without mathematics. Moreover, an adherent of this approach is also required to offer an explanation of why mathematics, even though dispensable, plays such a prominent role in science. The most influential hard road strategy in recent years has been Hartry Field's fictionalism. According to Field, mathematical sentences such as

$$\text{The unique prime factorisation of 255 is } 17 \times 5 \times 3 \tag{1}$$

are interpreted at face value. Thus interpreted such sentences imply the existence of mathematical entities[7] and so are literally false. He thus endorses

[4] Indeed, this is why Hartry Field [13, p. 4] suggests that the indispensability argument is the only non-question begging argument for mathematical realism, At least, it is no more question begging than standard arguments for scientific realism.

[5] Joseph Melia [17] claims that mathematical entities merely allow for more economical statement of theories; they do not simplify the theories in the right kind of way and they do not lend explanatory power to the theories in which they appear. While I think he is wrong about this (see [12]) I agree that there are some interesting issues to be explored here.

[6] Proposals along these lines include [2], [5, chap. 7], [10], and [17].

[7] Since it follows (in classical logic, at least) that prime numbers such as 17, 5 and 3, and composite numbers such as 255 exist. Of course, in free logic such conclusions do not follow from (1).

fictionalism about mathematics. Not all of the usually accepted "truths" of mathematics come out false though. Negative existential claims like 'there is no largest prime' and universally quantified sentences such as 'every natural number has a unique prime factorisation' are true according to the fictionalist. But they are vacuously true; they are true because there are no prime numbers and because there are no natural numbers respectively.

The Field-style fictionalist cannot rest there though. The fictionalist must show how our best scientific theories can be purged of their mathematical content and explain why mathematics can be used in empirical science without (crudely speaking) its falsity infecting the rest of the scientific theories. Field makes significant inroads on the former project by adopting a Hilbert-style geometric approach to Newtonian gravitational theory. On this approach, space-time points are compared with respect to their gravitational potential, for example, and this eliminates the need for gravitational potential functions [13]. Field then proves a representation theorem which demonstrates the adequacy of the approach. The project of explaining why the falsity of mathematics does not infect the rest of science is tackled by proving (and arguing for the plausibility of) a conservativeness result. The conservativeness result (if correct) shows that a mathematical theory M is conservative in the sense that for any body of nominalistic assertions N and any particular nominalistic assertion A, A is not a consequence of $N + M$ unless it's a consequence of N alone. With this in place, Field-style fictionalism is in a position to resolve the marital difficulties outlined above. One can coherently be both a scientific realist and a mathematical nominalist.[8]

2.2 Divorce I: Realism Gets the House

Another way of dealing with the tension outlined above is to move to a more thorough-going realism. One can hold onto one's scientific realist scruples and (perhaps reluctantly) admit that accepting inference to the best explanation and the realist package has some unforeseen consequences: one needs to be realist about a bit more than one initially bargained for. The realism extends to include all entitles indispensable to our best scientific theories, and these include at least some mathematical entities. This option is no reconciliation of scientific realism and mathematical nominalism. On this option, realism wins the day and nominalism is rejected.[9]

No divorce is so neat as this though. The mathematical realist still owes an account of the epistemology of mathematics and perhaps also an account of the nature of mathematical entities that jibes with that epistemology. After all, the indispensability argument, on the face of it at least, does not

[8] There are, of course, many further difficulties facing Field's project and many objections. I won't pursue such matters here. (See [9] for a good discussion of some of these.) At this stage I merely want to outline the various options.

[9] See [11] and [25] for defences of this approach.

tell us anything about either mathematical epistemology or the nature of mathematical entities.[10]

2.3 Divorce II: Instrumentalism Gets the House

Another option is to hold fast onto one's nominalist sensibilities and reject the form of argument that produced the tension in the first place. But as I've already pointed out rejecting the indispensability argument would seem to undermine a central plank of the scientific realist's platform. "So be it", you might say. "If mathematical realism is the price one pays for scientific realism, then the price is too high". According to this line of thought, anti-realism wins the day and it is mathematical realism that is rejected.

The mathematical nominalist is not home free though. It is not enough to simply reject the indispensability argument (and with it inference to the best explanation) and join the anti-realist camp. Consider, for example, what many take to be the most sophisticated anti-realist philosophy of science: Bas van Fraassen's constructive empiricism [29]. Constructive empiricism makes heavy use of mathematics in both its articulation and defence. Indeed, the crucial notion for constructive empiricism is that of empirical adequacy and this is spelled out in terms of models, structures, and isomorphic mappings—all of which are mathematical entities.[11] This problem for nominalising constructive empiricism has been raised by Michael Resnik [25, pp. 49–50]. Indeed, van Fraassen himself sees the problem and accepts the considerable burden of showing how constructive empiricism might be nominalised:

> I am a nominalist [...] Yet I do not for a moment think that science should eschew the use of mathematics. I have not worked out a nominalist philosophy of mathematics—my trying has not carried me that far. Yet I am clear that it would have to be a fictionalist account, legitimating the use of mathematics and all its intratheoretic distinctions in the course of that use, unaffected by disbelief in the entities mathematical statements purport to be about. [30, p. 303]

One option would be to embark on a Field style nominalisation project but this is not likely to be fruitful. As I pointed out earlier, Field utilises a

[10] As we'll see in section 4, I think that the indispensability argument does tells us quite a bit about the epistemology of mathematics. The indispensability argument, after all, does come with a holist epistemology, according to which we have knowledge of mathematical entities by the role they play in our best scientific theories. Moreover, this is no different from how we gain knowledge of other theoretical entities in science. See [10] (for example) for criticism of the holist epistemology that emerges from the indispensability argument and [3] for criticism of the indispensability argument's failure to say anything much about the nature of mathematical entities.

[11] A theory is empirically adequate if it has a model such that all appearances are isomorphic to empirical sub-structures of that model [29, p. 64].

Hilbert-style geometric approach to space-time. This involves quantification over space-time points and these are thus treated as real entities. This is something that many nominalists are unhappy about. But for constructive empiricists, realism about space-time points is out of the question. What other options are there then? Our constructive empiricist might employ one of the easy road strategies of section 2.1. But then it's not clear what constructive empiricism is bringing to the party. After all, if one of the easy road strategies of section 2.1 can be made to work, there was no need to retreat to constructive empiricism in the first place. Another option would be to reformulate the crucial notions of constructive empiricism—empirical adequacy and so on—in such a way as not to involve quantification over mathematical entities. This may well be possible but I leave the pursuit of such options for those with more sympathy for constructive empiricism.[12] In any case, this is most definitely not an option that a robust realist such as Musgrave is likely to find attractive!

3 Musgrave's Argument for Nominalism

Musgrave has entertained a couple of different approaches to the tension between scientific realism an mathematical nominalism. His first shot at a philosophy of mathematics was a version of if-thenism [18]. According to this view, mathematics consists of conditional statements such as 'If the conjunction of the Peano axioms, then there are infinitely many prime numbers'. Later, Musgrave defended a Field-style fictionalism [19]. Although these two approaches are rather different in detail they are similar in spirit. They are both nominalist philosophies of mathematics that only accept the truth of mathematical sentences once they are imbedded in a suitable construction such as a conditional ('if the conjunction of the Peano axioms then ...') or a fictional operator ('in the story of mathematics ...'). Rather than discuss the details of Musgrave's philosophy of mathematics, I want to consider his motivation for treating mathematical entities differently from other unobservable entities.

A central intuition that many nominalists have is that because mathematical entities are non-causal, they cannot make a difference to the way the physical world is.[13] If the existence of mathematical entities doesn't make a difference—that is, the physical world would be the same with or without mathematical entities—then there would seem to be no reason to believe in them. In his paper, 'Arithmetical Platonism: Is Wright Wrong or Must Field Yield?' [19], Musgrave explores this line of thought in an interesting and original way.[14] Instead of focussing on whether mathematical entities

[12] Otávio Bueno [7] and [8] has been doing some interesting work in this direction.

[13] See, for example, [1] for an articulation of this line of thought, and [4] for a reply.

[14] Musgrave suggests [19, pp. 90–91] that the argument I'm about to outline is just

make a difference to the physical world and what bearing this has on the epistemology of mathematics, Musgrave shifts the focus to the question of how we might falsify the hypothesis that there are mathematical entities.

> Imagine that all the evidence that induces scientists to believe (tentatively) in electrons had turned out differently. Imagine that electron-theory turned out to be wrong and electrons went the way of phlogiston or the heavenly spheres. Popperians think this *might* happen to any of the theoretical posits of science. But can we imagine natural numbers going the way of phlogiston, can we imagine evidence piling up to the effect that there are no natural numbers? This must be possible, if the indispensability argument is right and natural numbers are a theoretical posit in the same epistemological boat as electrons.
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> But surely, if natural numbers do exist, they exist of necessity, in all possible worlds. If so, no empirical evidence concerning the nature of the actual world can tell against them. If so, no empirical evidence can tell in favour of them either. The indispensability argument for natural numbers is mistaken. [19, pp. 90–91]

Musgrave, the scientific realist, argues that electrons make a difference to the way the physical world is. This means that the existence of electrons can be confirmed by crucial experiments such as the Millikan oil-drop experiment (and others).[15] But in the case of mathematical entities, Musgrave argues, it is difficult to see how any experiment could provide confirmation of their existence, and, we might add, their properties. The reason Musgrave gives is that mathematical entities, if they exist, exist of necessity, so their presence or absence cannot be established by appeal to crucial experiments. In short, the hypothesis that electrons exist can be falsified whereas the the hypothesis that mathematical objects exist cannot be falsified.

I take it that Musgrave's objection presents serious difficulties for any defender of the indispensability argument who takes mathematical entities to exist of necessity. There is, however, another position for the defender of the indispensability argument to adopt: the position that affords *contingent* existence to mathematical entities.[16] It might seem that this position isn't touched by the Musgrave objection, but I think Musgrave's concerns

another way of making Field's [13, pp. 11–12] point about conservativeness: mathematics does not need to be true to be good, it just needs to be conservative. I think Musgrave's argument is significantly different from Field's. At the very least, Musgrave's argument is different enough to warrant separate attention. I discuss Musgrave's take on Field's argument in [11, chap. 6].

[15] Indeed, experiments such as Millikan's yield crucial confirmations of not only the existence of electrons but also their mass.

[16] This is the position I endorse in [11]. Hartry Field [15] also accepts that the existence or non-existence of mathematical entities is a contingent matter, though he takes mathematical entities to contingently fail to exist.

here run a little deeper. Musgrave may be seen to be challenging the defender of contingently existing mathematical entities to provide the details of possible crucial experiments that might give us reason to accept or reject the existence of mathematical entities. I think this challenge can be met, though perhaps not in a fashion that will satisfy Musgrave. If I'm right about this, we have reached the source of Musgrave's nominalist sympathies and identified an important point of contention between nominalists and mathematical realists. So let me sketch how I take it that the challenge to provide crucial experiments for the existence of mathematical entities might be met.

According to the most plausible reading of the indispensability argument, mathematical entities exist contingently and the evidence for their existence comes from the confirmation of our best scientific theories (and the indispensable role mathematical entities play in those theories). Mathematical entities do not need to play causal roles in those theories (indeed, it is generally agreed that they do not play such roles). But if they do not play causal roles, what roles are left? Asking after a crucial experiment for the existence of an entity is akin to identifying a crucial causal connection of the entity in question. But at this point the Quinean simply digs her heals in and insists that there need not be any crucial experiment in the sense that Musgrave seeks. There will not be any experiment that directly confirms the existence of mathematical entities. This is not to say, however, that mathematical entities are without empirical support. According to the Quinean, mathematical entities are *indirectly* confirmed by whatever confirmation our best scientific theories enjoy.

But this doesn't address the issue of specifying the role of mathematical entities in these theories? Elsewhere [11, 12] I've argued that mathematics may contribute to the unificatory power and other theoretical virtues of scientific theories. We need to think of these theories holistically though. We need to resist any demand for crucial experiments—not just for mathematical entities, but for any entity. The thorough-going holist, would deny that even the Millikan oil-drop experiment is a crucial experiment. This experiment, after all, had auxiliary assumptions about the behavior of oil drops in gravitational fields and the behaviour of charged particles in electric fields, for instance. Such assumptions are not particularly controversial—that's not the confirmational holist's point. Their point is simply that a great deal more than the hypothesis in question is being tested and confirmed, even in so-called *crucial* experiments.

So, I claim that the source of Musgrave's inclination for trying to salvage the marriage of scientific realism and mathematical nominalism lies in his separatist (Popperian) confirmation theory. Separatist confirmational theories demand more than merely stating that some entity "plays a role in our best theory". The separatist wants a crucial experiment that identifies the causal roles of the entities in question. The confirmational holist, on the other hand, sees this latter request as simply unreasonable—at least in the context of establishing the existence of the entities in question.

Identifying the source of the disagreement is one thing, the real issue is surely that of determining which theory of confirmation is to preferred. Here, however, we have something of a nill-all draw. It's fair to say, that both Popperian falsification and Quinean confirmational holism find few supporters these days—most philosophers of science would not see either as a viable theory of confirmation. Be that as it may, the issues we've been concerned with do not depend so much on the fine details of these two theories of confirmation. After all, although Musgrave is a Popperian on such matters, the full details of Popper's philosophy of science were never invoked nor called into question. Musgrave's argument, it would seem, could be advanced on any separatist theory of confirmation.[17] Indeed, Elliot Sober [28] pursues much the same line of attack on the indispensability argument via another separatist theory of confirmation, namely his contrastive empiricism. And likewise, as I've argued elsewhere [11], the confirmational holist need not endorse the more radical holism of Quine. Still, there is a substantial issue sorting out whether any particular separatist or, alternatively, holist theory of confirmation can be made to fly. Obviously that is a large a task and one I cannot do justice to here. I'm content to identify a significant intuition that drives Musgrave (and others) in the exploration of issues concerning realism and nominalism in the philosophy of mathematics.[18]

I can't resist mentioning, however, the delicacy of the position the separatist finds himself in. The separatist needs to be able to avoid a couple of nearby slippery slopes. After all, if a crucial experiment must be devised for ever kind of entity to which we are to be ontologically committed, care needs to be taken about certain problem cases—those involving entities that scientific realists are committed to but which seem to lack crucial experiments. Some of the problem cases obviously involve unobservable entities such as electrons, quarks, black holes and the like. Typically the scientific realist is able to invoke the causal powers of such entities to design a crucial experiment. This is how the Millikan oil drop experiment worked. But what of unobservable entities that have causal powers but with which we have no contact? Consider, for example, stars and planets outside our own light cone. What are the crucial experiments that establish the existence of these entities? Of course there are responses to such problem cases, but the response must not license a slide to scientific instrumentalism or, alternatively, a slide to mathematical realism. Either slide would be to give the game away. For instance, I take it that the following response to the problem cases is illegitimate: we accept the existence of stars and planets

[17]Which is why I think, for present purposes, there is little point in criticising falsification.

[18]See [25, chap. 7] for a nice discussion of holism and its relevance the the realist–antirealist debate in the philosophy of mathematics.

outside our light cone because they play an indispensable role in our best cosmological theories. This won't do because (a) it violates the separatist criterion of providing a crucial experiment and (b) the appeal to playing an indispensable role (without further qualification) in a best scientific theory would also seem to license the acceptance of mathematical entities. In short, this response amounts to giving the game away to the mathematical realist. A similar unacceptable slide to anti-realism beckons, if our nominalist scientific realist decides to deny the existence of stars and planets outside our light cone. As I suggested above, there are options available here, but those wishing to salvage the marriage of scientific realism and mathematical nominalism need to be very careful about their treatment of such problem cases.

I now turn to a well-known epistemic argument used as a motivation for mathematical nominalism. I'll show how similar holist and separatist considerations impact on the ensuing debate in very similar ways as those outlined above.

4 Separatism and Holism about Justification

A great deal of the literature on the realism–antirealism debate in the philosophy of mathematics focusses on epistemology. In particular, nominalists typically take Paul Benacerraf's [6] epistemic challenge to mathematical realism as a challenge that cannot be met. Although Benacerraf originally presented his challenge in terms of the causal theory of knowledge, the essence of his argument can be captured without recourse to this now unpopular epistemology. Hartry Field puts the challenge as follows (emphasis in the original):

> Benacerraf's challenge—or at least, the challenge which his paper suggests to me—is to provide an account of the mechanisms that explain how our beliefs about these remote entities can so well reflect the facts about them. The idea is that *if it appears in principle impossible to explain this*, then that tends to *undermine* the belief in mathematical entities, *despite* whatever reasons we might have for believing in them. [14, p. 26]

As I've already mentioned, Field too is in favour of saving the union of mathematical nominalism and scientific realism. And from the above quotation we see that one of Field's motivations for defending nominalism is the epistemic problem for mathematical realism.[19] What's interesting here is that both Field and Musgrave agree that we should save the marriage

[19] Field has other motivations as well: the quest for intrinsic explanations and the elimination of arbitrariness from scientific theories. See [13, p. xi] for more on these issues.

in question, and they even agree on the best way to go about this: Field-style fictionalism. The difference is that Field is motivated (in part) by an epistemic problem for mathematical realism, where as, on the face of it at least, Musgrave is motivated by something else. In the last section I argued that Musgrave's motivation arises from separatist (as opposed to holist) sympathies about theory confirmation. But I think Musgrave's motivation has some interesting points of contact with Field's.

The usual construal of the Benacerraf-Field epistemic challenge is a challenge for the platonist to explain the reliability of mathematicians' beliefs. But implicit in this challenge is that the mathematical beliefs be taken *one at a time*. That is, the platonist must account for the reliability of the inference from 'mathematicians believe that P' to P. But put thus, the challenge assumes a separatist epistemology, according to which beliefs are justified one at a time. The epistemological holist will argue that this is wrong headed; beliefs are justified as packages. How does this help answer the Benacerraf-Field challenge? Well, if we drop the demand for justification of beliefs one belief at a time, then the mathematical realist can appeal to a holist epistemology to meet the challenge in question: we justify our mathematical beliefs by the role they play in broader systems of beliefs (namely, our best scientific theories).[20] Any dissatisfaction with such a holist response would seem to arise from separatist sympathies with regard to justification.

So Field is right that the main thrust of Benacerraf's epistemic challenge does not rely on the causal theory of knowledge. But by stating the challenge in reliabilist terms, Field still assumes a separatist epistemology. Moreover, it is precisely here that the holist will object. So once again we see that holist sympathies push towards "realism getting the house" whereas separatist sympathies push for "saving the marriage". This time the separatist and holist sympathies concern justification not confirmation, but clearly these two notions are closely related.

5 Conclusion

I've outlined some of the problems associated with reconciling scientific realism with mathematical nominalism. In the light of these problems it might be wondered why anyone would want to save this marriage. The first reason is that divorces are difficult: both of the divorce options I presented face substantial philosophical problems. This much is well known. Less appreciated, I think, is the role played by separatism and holism about both confirmation and justification. I've argued that, with respect to both justification and confirmation, separatist sympathies push for mathematical nominalism and holist sympathies push for mathematical realism. If this is

[20] See [11, p. 154], [26, chap. 3], and [27] for presentations of this response to the Benacerraf-Field challenge.

right, it seems that we have found a fruitful and appropriate place to focus our attention in attacking the realism–antirealism debate in the philosophy of mathematics.[21] I take this to be one of Musgrave's most significant contribution to the philosophy of mathematics. While my sympathies are with holism and mathematical realism, his with separatism and mathematical nominalism, we agree, I think, on how to approach the matter in question and on what some of the broader underlying issues are. In the philosophy of mathematics, at least, such agreement is non-trivial. But at the end of the day, I disagree with Musgrave about the prospects for saving the marriage of scientific realism and mathematical nominalism. I'm of the view that this marriage was never meant to be.[22]

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[21] Indeed, a great deal of contemporary work in the philosophy of mathematics is directed at questioning confirmational holism. See, for example, [16], [28], and, of course, Musgrave's contribution [19].

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