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Naturalism in Mathematics, by Penelope Maddy. Oxford: Clarendon Press, 1997. Pp. viii + 254. GBP32.50

This book represents the culmination of Penelope Maddy's recent work in the philosophy of mathematics. Here Maddy recants the mathematical realism she defended in *Realism in Mathematics* (Oxford: Clarendon, 1990) and argues instead for a somewhat Wittgensteinian anti-metaphysical position, at least with regard to the philosophy of mathematics. We thus see a dramatic change in her views on the ontology of mathematics, as well as a subtle but significant change in her approach to metaphysics generally. These differences, however, should not overshadow the continuity this book has with Maddy's previous work: her central concern has always been the vindication of the pursuit of independent questions in set theory and their bearing on new set theoretic axiom candidates. She continues her investigation of this important topic in *Naturalism in Mathematics*, albeit with a different philosophical outlook. The book is beautifully written, tightly argued and makes compelling reading. I believe the position Maddy introduces and defends—*set theoretic naturalism*—is a significant and original addition to the philosophy of mathematics landscape, and one that will certainly attract a great deal of attention. Despite misgivings about some of Maddy's central theses, I am in no doubt as to the importance of this work and I find I have considerable sympathy with her concerns, if not her final position.

Ultimately Maddy wants a philosophical account of mathematics that squares with mathematical practice and to this end she considers the question of the appropriate extensions to standard axiomatic set theory (ZFC). That is, she is concerned with the question of what grounds we have for accepting or rejecting new axiom candidates such as Gödels axiom of constructibility, $V = L$, or a large cardinal axiom such as MC (there exists a measurable cardinal). It turns out that there is a certain amount of agreement amongst set theorists that $V \neq L$ and that some large cardinal axiom or other (not necessarily MC) is the right way to proceed. According to Maddy, a philosophical account of mathematics that does not vindicate this preference is implausible, and so the question of new ZFC axioms provides a crucial test for the philosophy of mathematics.

Maddy begins by considering the origins of set theory and the evidence put forward for the standard ZFC axioms—including the once controversial Axiom of Choice. She then notes that there are questions such as the continuum hypothesis—Does $2^{\aleph_0} = \aleph_1$?—and questions about the measurability of certain well-defined sets of reals in the projective hierarchy that are independent of ZFC. That is, both the negative and positive answers are consistent with ZFC (provided ZFC itself is consistent). A great deal of effort in contemporary set theory is devoted to settling such questions. Clearly they cannot be settled by proving theorems—they are to be settled by finding axioms that extend ZFC in a natural and appropriate way. Moreover, set theorists argue over what extensions are natural and appropriate. The obvious way to make sense of this debate is to follow Gödel and appeal to mathematical realism. On this view, the set theorists are trying to find the correct axiomatisation of an independently existing universe of sets. Indeed, this was the primary motivation for Maddy's aforementioned realism. However, motivation is one thing, justification another entirely, and Maddy, like many other mathematical realists, explicitly appealed to the Quine-Putnam indispensability argument for the latter. She now subjects this argument to much closer scrutiny than she did in her previous book.

The Quine-Putnam indispensability argument purports to deliver the conclusion that (at least some) mathematical entities exist, from the premises of naturalism, confirmational holism and the indispensability of mathematical entities to natural science. It runs something like this: (i) we ought to believe our best confirmed scientific theories and, in particular, we ought to believe in the existence of entities that are indispensable to those theories (naturalism); (ii) our best scientific theories are confirmed as wholes (holism); (iii) mathematical entities are indispensable to our best confirmed scientific theories (indispensability); therefore (iv) we ought to be committed to the existence of mathematical entities and, moreover, their existence is supported by exactly the same empirical evidence as supports the existence of other theoretical entities of science.

Maddy is now convinced that the indispensability argument does not work. She presents a number of reasons for this, all of which revolve around tensions between holism and naturalism. For example, if it turns out, as some quantum physicists have suggested, that space-time is discrete, continuum mathematics (such as calculus) may find itself without any non-idealised applications in physical science. According to the indispensability argument that would mean much of the evidence for realism about the continuum is eroded. Maddy claims that if the indispensability argument is correct, set theorists ought to be keeping a close eye on developments in quantum mechanics in order to help settle some of the open questions in set theory. Given that set theorists do not seem to do this, the indispensability theorist is apparently committed to suggesting serious revisions of mathematical methodology:

> [T]he vicissitudes of applied mathematics do not seem to affect the methodology of mathematics in the way that they would if applications were in fact the arbiters of mathematical ontology. And this means that mathematics [...] seems not to be conducted as it would be conducted if the presuppositions of our indispensability theorist were correct. (p. 159)

She goes on to consider the move of suggesting that the set theorists in question are in error and that they ought to revise their methods. She concludes:

> My own inclination—and here I follow Quine himself [...]—is to reject such moves. This simple inclination lies at the heart of naturalism. (p. 160)

Although I disagree with Maddy's assessment of the deficiencies of the indispensability argument, this is a substantial issue and one which I cannot do justice to here. (Cf. my 'In Defence of Indispensability' (*Philosophia Mathematica 3*, Vol. 6, No. 1 (1998) pp. 39–62) for details.) Instead, I'll say a little about the related matter of Quine's endorsement of $V = L$. Maddy points out that

> Quine counsels us to economize, like good natural scientists, and thus to prefer $V = L$, while actual set theorists reject $V = L$ for its miserliness. (p. 131)

And that

> Quine's application of indispensability considerations has led him to a stand (on $V = L$) precisely opposite to that of the set theoretic community. (p. 106)

This disagreement between Quine and the set theorists is enough for Quinean indispensability theory to seem implausible to Maddy. It should not be forgotten, however, that Quine has company here. In particular, other set theorists such as Keith Devlin endorse $V = L$ (cf. *The Axiom of Constructibility*, Berlin: Springer, 1977). This is not simply a case of a philosopher disagreeing with the set theory community on philosophical grounds. This debate is perhaps best seen as a scientific dispute within set theory (with Quine and Devlin admittedly on the minority side) over either the weighting of conflicting goals of set theory or what the relevant goals of set theory are. Maddy's tendency to portray Quine as opposing the set theorists on philosophical grounds can be somewhat misleading (indeed, there is more than a hint of Quine practicing first philosophy in it!).

Secondly, it is far from clear that *indispensability considerations* are responsible for Quine's endorsement of $V = L$. After all, he explicitly gives "considerations of simplicity, economy, and naturalness" as his reasons for preferring $V = L$, because "[$V = L$] inactivates the more gratuitous flights of higher set theory" (Quine, *Pursuit of Truth*, revised edition, Cambridge, MA.: Harvard University Press, 1992, p. 95). This endorsement seems to have more to do with Quine's well-known taste for desert landscapes than with indispensability considerations. Although these two are closely related in Quine's thinking, it seems that there is room for a defender of the indispensability argument to prefer MC, say, over $V = L$ on the grounds of the unification and expressive power the former brings to science as a whole. The indispensability theorist may well admit that the inflated ontology that MC brings is a cost, but that it is a cost worth incurring. Although such a view is clearly not endorsed by Quine, it is not ruled out by indispensability considerations alone. In short, I think Maddy's quarrel here is with Quine's zeal for simplicity, *not* with the indispensability argument itself.

In any case, Maddy's rejection of the indispensability argument means she no longer sees realism as a viable option, and she is forced to look further afield for a philosophy of mathematics that sits well with mathematical practice—a philosophy of mathematics that will legitimise set theorists' interests in independent questions. This she finds in *set theoretic naturalism*.

> What I propose [...] is a mathematical naturalism that extends the same respect to mathematical practice that the Quinean naturalist extends to scientific practice. [...] [T]he mathematical naturalist [holds] that mathematics is not answerable to any extra-mathematical tribunal and not in need of any justification beyond proof and the axiomatic method. (p. 184)

For Maddy, then, the methodology of mathematics is in no need of justification external to mathematics. In particular, it does not depend on the role mathematics plays in empirical science for its legitimacy (as it does in the Quinean picture). Moreover, issues of realism and anti-realism are irrelevant to Maddy's project—what is important is that the legitimacy of investigating the continuum hypothesis, say, is ensured. As I've already noted, it was her interest in such issues that originally drove her to mathematical realism, and now she has found a way to preserve the desirable methodological consequences of realism without relying on what she sees as a bankrupt metaphysical position. (This is the Wittgensteinian turn in her approach to metaphysics that I mentioned earlier.)

Finally, Maddy turns her attention to the support particular set-theoretic axiom candidates receive from her new naturalistic perspective. In particular, she argues against $V = L$ and succeeds, I think, in placing another nail in the coffin of this much-discussed axiom. A major part of this task is what she calls the "boundary problem". This is the problem of distinguishing between legitimate intra-mathematical debate over a new axiom candidate and extra-mathematical (either philosophical or scientific) debate. Her discussion here (pp. 188–193) is enlightening, irrespective of what you think of the rest of her project, as it bears directly on the more general problem of distinguishing between first philosophy and naturalised philosophy.

In summary, this is a very important book covering some fascinating terrain on the border between philosophy and mathematics. The book will thus be of considerable interest to philosophers of mathematics and mathematicians alike. Indeed, Maddy's engaging style and clear explanations of the more technical material makes this book, like her last, accessible and of interest to the non-specialist as well. *Naturalism in Mathematics* should enjoy a wide readership and it will no doubt promote fruitful debate on the many important issues it raises.

MARK COLYVAN

School of Philosophy
University of Tasmania
GPO Box 252-41
Hobart, Tasmania, 7001
Australia