Morton, Adam and Stich, Stephen P. (eds.), *Benacerraf and his Critics* (Oxford: Blackwell, 1996) pp. xi, 271, GBP45.00 (cloth).

This is the latest in Blackwell's 'Philosophers and their Critics' series, featuring eleven new essays by various authors on Paul Benacerraf's philosophy. The essays are arranged into five sections: 'Platonism and Mathematical Truth', 'Indeterminacy Arguments', 'Logicism', 'Mathematics and Language', and 'Infinity' and cover all aspects of Benacerraf's published work (and PhD dissertation!). Contributors include George Boolos, John Earman and John Norton, Richard Grandy, Richard Jeffrey, Jerrold Katz, Penelope Maddy, Adam Morton, Robert Stalnaker, Mark Steiner, Steven Wagner and Paul Benacerraf himself. It is difficult to do justice to all the essays in a review such as this, so I shall confine my attention largely to a few key essays that directly address Benacerraf's two land-mark papers in the philosophy of mathematics: his 1965 'What Numbers Could Not Be' (reprinted in Benacerraf, P. and Putnam, H. *Philosophy of Mathematics*, second edition, 1983, pp. 272–294, henceforth *WNCNB*) and his 1973 'Mathematical Truth' (also reprinted in Benacerraf, P. and Putnam, H. *Philosophy of Mathematics*, second edition, 1983, pp. 403–420, henceforth *MT*). These two papers have been pivotal in discussions of the epistemology and ontology of mathematics in the past 25 years.

In *WNCNB* Benacerraf set out to undermine the prevailing view that numbers could be reduced to sets. This he did by pointing out that there is no unique reduction of the natural numbers to sets. Benacerraf then considered possible reasons for preferring one reduction over others, before deciding that there are no such reasons. He thus concludes section II of *WNCNB* with the claim that numbers could not be sets at all.

In his essay 'What Mathematical Truth Could Not Be – I', which opens the present volume, Benacerraf discusses the arguments of *WNCNB* (amongst other things). Indeed it is very interesting to read Benacerraf's own view on this important paper and how it has weathered the last 31 years. He notes that (p. 25):

> Most of the issues that are raised in the above argument [the argument of sections I and II of *WNCNB*] have received considerable discussion; very few, if any, have been satisfactorily resolved.

He then considers the weaknesses of the argument. The first is that many people (including Boolos, Dummett and others) feel that the Frege-Russell account or the Frege account of numbers as sets did not receive due consideration in *WNCNB*, and that one of these two is indeed the best candidate for the natural numbers. The second is that, in Benacerraf's words, 'no red-blooded realist [...] should accept the bald statement that if there isn't some *a priori* proof that some particular candidate reduction is the correct one, there can't be a "correct" one' (p. 26).

The third weakness concerns what we ought to conclude from a successful reduction of entities of one type to entities of another, and it's this weakness that I personally find most interesting. The argument of sections I and II of *WNCNB* is primarily directed at reductionists who are not inclined to deny the existence of entities that have been successfully reduced to entities of another kind. Benacerraf likens this type of reduction to packing for a trip to the tropics in which your taking of only one sweater is not to deny the existence of the others you own, it's just that you won't need them. That is, by reducing the natural numbers to some set theoretic structure, all that is to be concluded is that we could do without the numbers since we have an isomorphic copy of them. The numbers are not ruled out of the catalogue of the 'furniture of the universe' by such a reduction. Compare this with the thoroughgoing Ockhamist (such as Quine or Field) whose test for inclusion in the aforementioned catalogue is whether the entity in question is necessary for our best theories of the world (considered holistically). The argument of sections I and II of *WNCNB* does not have any impact on this latter position since 'the holistic Ockhamite [...] is not beholden to any notion of "getting it right" that transcends the best theory' (p. 27). As Benacerraf puts it (on behalf of Quine) 'why demand more than sets, if sets is all you need [...]?' (p. 28). These three weaknesses suggest three lines of attack on the argument of *WNCNB*, and of these Benacerraf has most sympathy with the first. That is, defending the Frege-Russell analysis of natural number as the 'right one'.

In the third section of *WNCNB* Benacerraf took matters further and argued for the conclusion that 'not only could numbers not be sets, they couldn't even be numbers' (p. 23). Benacerraf points out in his essay in this volume that section III of *WNCNB* has been much misunderstood. The conclusions of section III were being advanced with much less confidence than those of the previous two sections, as they depend on (i) the structuralist premise that arithmetic is the science of progressions (not a particular progression, but progressions in general), and (ii) the claim that no (particular) system of objects could satisfy (i). Both of these are obviously controversial.

Benacerraf concludes his discussion of *WNCNB* in 'What Mathematical Truth Could Not Be – I' by noting that: 'I see *WNCNB* as much less of a "definitive" statement on the issues it addresses th[a]n I imagine I once did, either in its first two sections, or in its more tentative, but also more sweeping

Section III' (p. 28). He then goes on to discuss the use of metamathematical results to gain philosophical conclusions. This is a tactic used in *WNCNB*, for example, where the metamathematical result that the natural numbers have more than one representation in set theory, is used to gain the philosophical conclusion that the natural numbers can't be sets at all. His discussion here is also very interesting. He sounds a warning to all philosophers influenced by metamathematical results, be they Gödel's incompleteness theorems, the Löwenheim-Skolem theorem or the indeterminacy thesis used in *WNCNB*. Benacerraf's point here is a simple but important one. In order to argue for a philosophical conclusion from some metamathematical result we should be extremely careful how we proceed, for any such argument cannot rest on the metamathematical result alone. There must be some philosophical premise also supplied, and this is often illicitly smuggled through the back door.

Benacerraf's discussion of metamathematical results in philosophical arguments is reinforced by Jerrold Katz' discussion of the indeterminacy argument of *WNCNB* in his essay 'Skepticism about Numbers and Indeterminacy Arguments'. Katz compares Benacerraf's argument with two very influential indeterminacy arguments in the philosophy of language, namely Quine's argument for the indeterminacy of translation (Quine, W.V. *Word and Object*, 1960) and Kripke's rule following argument (Kripke, S. *Wittgenstein on Rules and Private Language*, 1982). Katz claims that all these arguments share the same structure and that all are 'irredeemably flawed' (p. 119). All three arguments incorporate the following two claims: (i) there is more than one entity that satisfies the requirements of the task at hand (sequences of objects in Benacerraf's case, rules in Kripke's case and translations in Quine's case) and (ii) there is no way to know which of these entities is the intended or 'correct' one. That is, the rival entities in question are functionally equivalent and yet, for an indeterminacy argument to have any force, they must be *incompatible*. Katz' solution is to point out that in order for us to recognise the incompatibility of the rival entities we must be able to recognise some feature that distinguishes them. We then use whatever this feature turns out to be as a way of resisting the alleged functional equivalence. In the Benacerraf case this means using non-structural properties of numbers to distinguish them from other sequences of entities.

I find Katz' discussion of the similarities between these three influential arguments of considerable interest and it does, as Katz claims, enable one to see all these arguments from a new, more abstract perspective. Furthermore, I find his approach to resisting the sceptical conclusions of these arguments promising, if less than entirely convincing, but here is not the place to discuss such matters in detail.

Benacerraf's other major contribution to the philosophy of mathematics was his paper *MT*. Here he proposes the following challenge to philosophers of mathematics: (i) to naturalise mathematical epistemology, and (ii) to produce a semantics for mathematics that meshes with that of the rest of language. On a Platonist account of mathematics the second challenge is met easily since a proposition such as 'there exists a prime number greater than 3' is made true by the existence of the number 5 (amongst others), just as 'there exists a city larger than Melbourne' is made true by the existence of the city New York (amongst others). The problem for Platonism, however, is to provide a naturalised account of mathematical epistemology. Benacerraf also showed how various anti-realist views of mathematical entities meet the first challenge but not the second. As Penelope Maddy points out though, in her essay 'The Legacy of "Mathematical Truth" ', it was the challenge to Platonism that *MT* is best remembered for.

The challenge to Platonism in *MT* is an intriguing argument, since it seems to have survived the loss of consensus on both its premises—the causal theory of knowledge and the correspondence theory of truth! In Maddy's words 'there is a certain undeniable fascination to a problem so resilient!' (p. 64). Indeed, it is interesting to try to ascertain what it is that gives this argument its longevity. Perhaps the essence of Benacerraf's problem for Platonism is, as Hartry Field suggests in the introduction to *Realism, Mathematics and Modality* (1989), that of explaining the *reliability* of our mathematical beliefs.

Undoubtedly the greatest shortcoming of the current volume is that Benacerraf's own thoughts on the reasons for the resilience of the argument of *MT* are not presented. (Apparently he had intended to discuss such matters in his contribution 'What Mathematical Truth Could Not Be – I' but that essay grew too long and so the discussion of *MT* was left for a sequel.) This omission is particularly disappointing given the, at best, tangential relevance of a couple of the essays in this volume. It would also have been good to see an essay by Hartry Field on *MT*, since his presentation of the problem for Platonism (mentioned above) is taken by many to be the best statement of Benacerraf's 1973 challenge since the demise of the causal theory of knowledge. So much for my quibbles about what's *not* in the volume, let me return to Maddy on *MT*.

After discussing the demise of the two explicit premises of the argument against Platonism in *MT*, Maddy looks at the implicit role indispensability theory plays in Benacerraf's argument (as the primary argument for Platonism). (This theme is also taken up by Steven Wagner in his paper in this volume, 'Prospects for Platonism'.) Maddy, however, has serious reservations about the soundness of indispensability arguments (arguments that go from the indispensability of some entity to our best scientific theories, to the existence of that entity) based on attention to scientific and mathematical practice. This leads her to question the relevance of Benacerraf's challenge to Platonism (p. 67):

> Despite its impressive immunity to attacks on its explicit premises, the Benacerrafian challenge to Platonism becomes irrelevant in the absence of a strong argument for Platonism.

If her worries about the indispensability argument are sustained, and if there is, as Maddy believes, no other good argument for Platonism (something which Wagner denies in his essay), then a reassessment of much of the philosophical work inspired by Benacerraf's challenge is necessary. For Maddy this has meant looking at why Platonism is so appealing. Here she agrees with Benacerraf, Gödel and others that Platonism legitimates the pursuit of independent questions in mathematics (such as Cantor's continuum hypothesis), in a way which is hard to legitimate otherwise. Nevertheless, Maddy argues that careful attention to, and respect for, mathematical methodology can also legitimate such pursuits (such as in her 'Set Theoretic Naturalism', *Journal of Symbolic Logic* Vol. 61, 490–514). She thus concludes that the enduring legacy of MT is the recommendation for more careful attention to actual scientific and mathematical practice. One finds it hard to disagree with such a sentiment.

Finally to typographical matters. There are numerous typographical errors, most of which are harmless. A few, however, are not and deserve mention. On page 26, line 1 read '=' for '5'. On page 43, line 19 read '$\vdash$' for '+'. On page 116, the last line should begin 'tion that is false if and only if it is not)'. On page 223 in the last line of the proof read 'δ_{100}' for 'δ'.

Other errors I noticed are, on page 225 the third line of the program is ambiguous. I take it that it should read:

> IF there is no such A and no such B THEN STOP at YES.

There also appears to be a missing line in that same program. I suggest that the following should be inserted between the 4th and 5th lines:

> IF there is such a B but no such A THEN STOP at NO.

None of these shortcomings, however, detracts in any significant way from a very interesting book addressing important philosophical topics.

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