Marcus Giaquinto. **The search for certainty: a philosophical account of foundations of mathematics.** Clarendon Press, Oxford 2002, xii + 286 pp.

There was a time when mathematical knowledge was thought to be certain—indeed, mathematical knowledge was seen as an exemplar of the kind of certainty that we would like for other areas of enquiry. To use a familiar foundationalist image, mathematical knowledge was seen as the secure base on which we could construct our epistemic edifices. Unlike empirical knowledge, mathematical knowledge was not thought to be subject to the kind of error that leads to revision and replacement of scientific theories. Once proven, a mathematical theorem was thought to stand for all time. But such faith in mathematics was shaken around the end of the nineteenth century when, despite a couple of hundred years of significant advancements in the rigorization of analysis, several set-theory paradoxes came to light that shook mathematics to its core. These paradoxes and the responses to them is the subject of Marcus Giaquinto's very engaging book.

The book presents an historically informed account of the mathematical and philosophical community's struggle with the crisis brought on by the paradoxes. The reader is taken on a fascinating tour of "one of the most brilliant intellectual explorations ever" (p. vii). The book is divided into 6 sections, each typically consists of several chapters. The first section, provides some of the nineteenth century historical background. In particular, the quest for rigorization of analysis is discussed. Section II presents three of the paradoxes—the Burali-Forti paradox, Cantor's paradox and Russell's paradox. The early responses by Cantor, Frege and Russell are discussed and the inadequacies of each response is revealed. The next section presents a few of the definability paradoxes such as Berry's paradox and the well known paradoxes of truth, such as the strengthened liar paradox and cyclical liar paradoxes. It also gives a detailed account of the Russell and Whitehead response to the paradoxes in terms of the vicious circle principle and *Principia Mathematica*'s theory of types—both simple type theory and the ramified theory. Ramsey's defense of logicism as a response to the paradoxes is also discussed in this section. Section IV focusses on Zermelo's axiomatic set theory and Hilbert's formalist/finitist program. The next section presents Gödel's incompleteness theorems and how these serve to undermine Hilbert's program. The final section defends the axiomatic approach to set theory as an adequate response to the paradoxes and draws some philosophical conclusions from the preceding discussion.

The take home lesson of this book is that the crisis in the foundations of mathematics is over. There is no good reason to believe that standard set theory—ZFC—is inconsistent. Moreover, this theory can be defended against the charge of ad hocness. In particular, ZFC has shed its last vestiges of (ad hoc) limitation of size axioms. The last such vestige being the axiom of replacement, and this can be dispensed with by adopting Levy's Reflection Principle. The set theory delivered, though, is a long way from the traditional idea of a foundation. Traditionally conceived, foundations are supposed to be self evident and secure—secure enough to serve as a basis on which other beliefs may rest. But the first move away from such a foundation came with Russell who, in the light of the paradoxes, claimed that axioms may be justified by the theorems they yield: "we tend to believe the premises because we can see that their consequences are true, instead of believing the consequences because we know the premises to be true" (Russell, B., 'The Regressive Method of Discovering the Premises of Mathematics' reprinted in D. Lackey (ed.) *Essays in Analysis*, George Allen and Unwin Ltd., 1973). This view was later endorsed by Gödel, and contemporary debates over large cardinal axioms serve to reinforce this "top-down" style of justification of the axioms (see Maddy, P., *Naturalism in Mathematics*, Oxford University Press, 1997). Such "top-down" justification suggests that set theory no longer serves as an epistemological foundation for mathematics—at least not in the traditional sense of 'foundation'. Indeed, this style of justification makes much more sense in the context of coherence epistemologies. The move from foundational epistemology to more holistic approaches strikes me as one of the legacies of the paradoxes. Traditional, foundational epistemology is just not tenable in these post-paradox times.

Although much of the material covered in this book will be familiar to philosophers of mathematics and philosophical logicians, Giaquinto is able to shed new light on many issues and he always gives a very sympathetic reading of the various views he discusses. Indeed, I think Giaquinto has written a very good book, though, one that's somewhat difficult to categorize. It professes not to be a history of twentieth century developments in the foundations of mathematics—it's stated goal is to "set out and engage with the main philosophical ideas and arguments" (p. vii). There is no doubt that it achieves its stated goal but it does this in an historically informed way. There is also a great deal of technical detail in this book. It is not, however, a book on mathematical logic (although it would make an excellent companion to a standard course on mathematical logic or philosophy of mathematics). Giaquinto prefers not to skimp on technical detail. He believes that to understand and engage with the issues in question one must be familiar, at least in broad brush strokes, with some of the key technical results. I couldn't agree more. But importantly, in providing the relevant technical details, Giaquinto does not presuppose much by way of logical and mathematical background. Indeed, one of the many admirable features of this book is that, without "dumbing down" the content, Giaquinto manages to provide an accessible account of some fairly technical material. I'd suggest that a dedicated reader with only a first course in logic or some undergraduate mathematics behind him or her would be able to benefit from this book.

It should also be clear by now that this is an ambitious book. It covers a great deal of territory and discusses many interesting issues, and, as you might expect, there are many interesting issues worthy of further discussion. Let me focus on just a couple of these here. The question of why we should pursue a uniform treatment of the paradoxes is very important and, in my view, warrants further attention. There is a strong intuition that the language paradoxes and the set theory paradoxes must be treated in much the same way. But this is so only if you think that the language paradoxes are at one with the set theory paradoxes. Opinion is divided on this latter issue. Ramsey argued that there was a significant difference between the two types of paradox, whereas Russell thought they were essentially the same. Although Giaquinto does discuss this issue (pp. 104–106), I think that there is much more to be said. In particular, it would have been good to have presented the details of some of the formal frameworks that (allegedly) demonstrate the common structure of the language and set theory paradoxes. One important contemporary framework is Graham Priest's inclosure schema (see Priest, G., *Beyond the Limits of Thought*, 2nd edition, Oxford University Press, 2002, especially pp. 133–136, for details).

A closely related issue which Giaquinto does not broach is the significance of Yablo-style paradoxes. Yablo's paradox is a liar-like paradox (though Yablo-style paradoxes now come in both language and set theory versions—see Sorensen, R.A., 'Yablo's Paradox and Kindred Infinite Liars', *Mind* 107 (1998): 137–55, for a survey and discussion of some of these) that, arguably, does not involve self reference or cycles of reference. If this, is right, then simple bans on circularity will not disarm this paradox. So, in order to advance a uniform solution to the paradoxes, we need to answer the question of where Yablo's paradox fits into the picture. It would be interesting to hear Giaquinto's thoughts on this matter.

These couple of comments are not intended to signal serious shortcomings of the book; an ambitious book such as this is bound to leave some issues unresolved. These are just some of the issues that I was led to think about while reading this excellent book.

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