JAMES ROBERT BROWN

*Philosophy of Mathematics: An Introduction to the World of Proofs and Pictures*.

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'Anyone sincerely interested in philosophy must be interested in the nature of mathematics, and I hope to show why. As for those who persist in thinking otherwise—let them burn in hell.' (xii)

There are at least two approaches to writing an introductory book on some area of philosophy. The first is to strive for a sense of objectivity and resist the temptation to take sides on key issues. Indeed, this is by far the most common approach and it has a lot to commend it. The other approach is a little more adventurous. It involves the author letting his or her readers know what he or she thinks about the issues in question and defending those views—as one would in a research work. With this approach the reader gets to see philosophy in action. For my money, I'm very pleased that Brown took the second option when writing this delightful book. The book is filled with Brown's insightful views on many issues in the philosophy of mathematics and, most importantly, his love of, and enthusiasm for, his subject is apparent on every page. You might not agree with all Brown has to say in this book, but there's no chance you'll fail to engage with the subject. Students, I think, will be provoked by Brown's up-front approach and inspired to think seriously about the philosophy of mathematics themselves.

Since the book is intended as both an introductory text and a vehicle for defending some controversial views about mathematics, Brown finds himself engaged in a delicate balancing act. This balancing act, I hasten to add, is performed to perfection. Brown manages to mount a sustained defence of his two central theses: mathematical Platonism and the thesis that pictures can serve as rigourous proofs. Moreover, Brown sees important connections between these two theses; he argues that picture proofs help Platonism answer the standard epistemic objections. As Brown puts it: 'some "pictures" are not pictures, but rather are windows to Plato's heaven' (39). But Brown also manages to introduce the reader to the usual topics of any standard introductory course on the philosophy of mathematics: intuitionism, formalism, structuralism, the Quine–Putnam indispensability argument, Benacerraf's epistemic challenge to Platonism, and so on. These latter topics, however, are presented in relation to the main theme of the book, not, as is so often the case, as isolated topics. The book thus has a coherence that is rare in introductory treatments of the philosophy of mathematics.

The book is also refreshing in other ways. There are a number of topics that this book addresses that are not usually covered in introductory books on the philosophy of mathematics. (Indeed, some of these topics are rarely discussed anywhere, but I dare say that will change.) These topics include the nature of applied mathematics, the role of definitions in mathematics, the role of proof in mathematics. All these topics are illustrated with fascinating and accessible mathematical examples from various branches of mathematics including: knot theory, graph theory, analysis, and number theory. Indeed, the examples are interesting in their own right and give the reader a sense of the diversity of techniques and subject matters that modern mathematics encompasses. The examples are no mere decorations though. They are always employed to illustrate the point at hand. For example, the discussion of the distribution of Mersenne primes and perfect numbers (160–4) beautifully illustrates both the difficulties and legitimacy of inductive inferences in mathematics. And the discussion of Conway notation (84–6) in knot theory is a wonderful illustration of the power and importance of notation in modern mathematics.

As I've already mentioned, one of the central theses defended in the book is that, contrary to accepted wisdom, pictures are not merely pedagogical aids, in mathematics—they can, in some instances, be legitimate and rigourous proofs. A defence of this thesis obviously requires examples of picture-proofs. The wealth of accessible and interesting examples that Brown calls upon here is a highlight of the book. He has some beautiful examples of pictures that he suggests prove the relevant theorems. Included here are well known examples, such as: the picture-proof of the intermediate-value theorem of calculus; the Greek picture-proof of $\sum_{j=1}^{n}(2j-1) = n^2$; and an example of a picture-proof of a fixed-point theorem of analysis. There are also many lesser known examples, as well as some very interesting examples of pictures that lead us astray.

Of course you can't do everything in a book this size, so some of the topics that you might normally expect to have star billing in an introductory treatment are relegated to supporting roles. For example, the discussion of Paul Benacerraf's epistemic problem for Platonism ('Mathematical Truth', Journal of Philosophy, Vol. 19, 1973, 661–79) is given rather short shrift. Brown shows how this epistemic-access problem relies on the causal theory of knowledge and then he demonstrates the inadequacy of this as an epistemology. He concludes his discussion (18): 'Once the causal theory is rejected, there is no objection to our knowing about abstract entities without being related to them. The problem of access is a pseudo-problem; resistence to Platonism is motivated by misplaced scruples.' No mention is given of what many philosophers consider the most compelling formulation of this problem: Hartry Field's presentation in the introduction to *Realism Mathematics and Modality* (Oxford: Blackwell, 1989, 25–30). This latter presentation does not rely on the causal theory of knowledge and so seems immune from Brown's rebuttal. Still you can't do everything and you can't please everyone. And all things considered, I'm pleased that

Brown covered the topics he did, even if sometimes it was at the expense of a deeper coverage of some such as this.

All in all this is a wonderful introduction to the philosophy of mathematics. It's lively, accessible, and, above all, a terrific read. It would make an ideal text for an undergraduate course on the philosophy of mathematics; indeed, I recommend it to anyone interested in the philosophy of mathematics—even specialists in the area can learn from this book (I certainly did).


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